# Toward an AdS/cold atom correspondence

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# Plan

- Unitarity regime
- Schrödinger symmetry: nonrelativistic conformal invariance
- Geometric realization of Schrödinger symmetry
- Green's functions in vacuum
- Many-body physics
- Conclusion

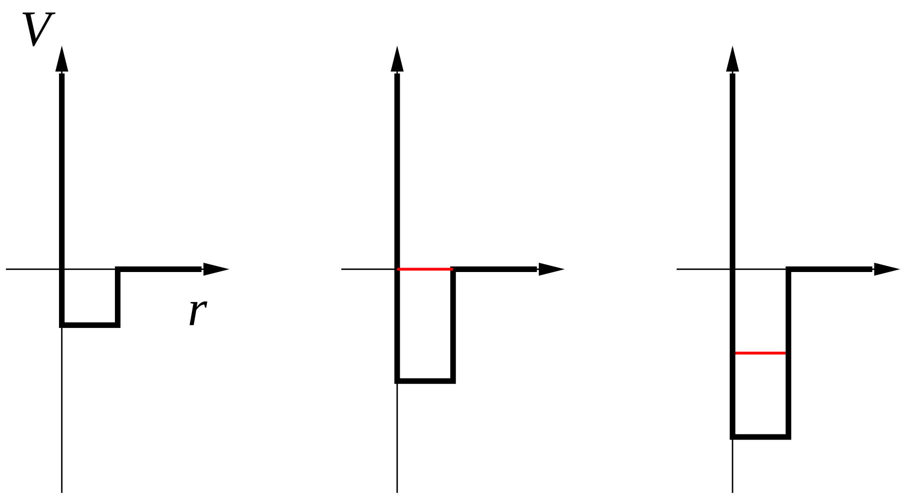
Collaborators: Y. Nishida, M. Rangamani, S. Ross, E. Thompson

Refs.: Y. Nishida, DTS, 0706.3746 (PRD)
Balasubramanian and McGreevy, 0804.4053 (PRL 2008)
DTS, 0804.3972 (PRD 2008)
Rangamani, Ross, DTS, Thompson, arXiv:0811.2049

# Unitarity regime

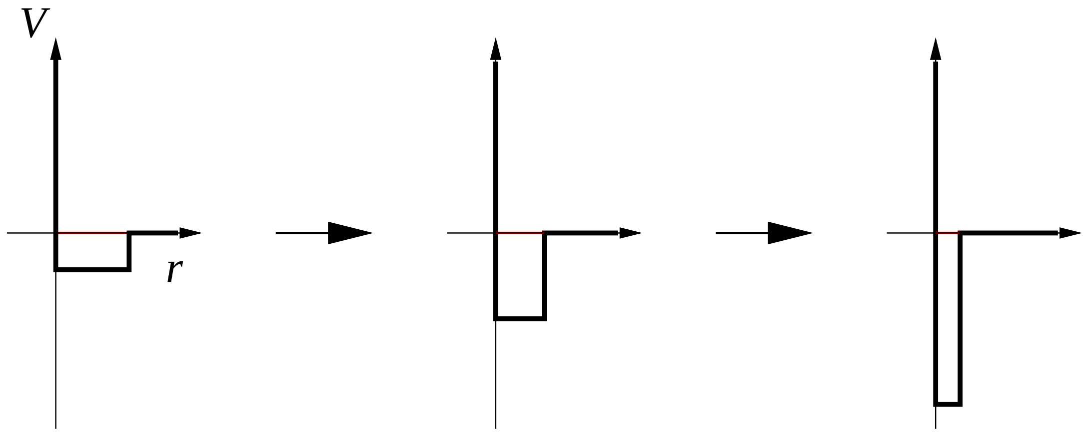
Consider a BCS-BEC crossover.
Assume a square-well potential, with fixed but small range $r_0$.

- $V_0 < 1/mr_0^2$: no bound state
- $V_0 = 1/mr_0^2$: one bound state appears, at first with zero energy
- $V_0 > 1/mr_0^2$: at least one bound state



# Unitarity regime (II)

Unitarity regime: take $r_0 \to 0$, keeping one bound state at zero energy.




In this limit: no intrinsic scale associated with the potential

In the language of scattering theory: infinite scattering length $a \to \infty$

$s$-wave scattering cross section saturates unitarity

# Boundary condition interpretation

Unitarity: taking Hamiltonian to be free:

$$H = \sum_i \frac{\mathbf{p}_i^2}{2m}$$

but imposing nontrivial boundary condition on the wavefunction:

$$\Psi(\underbrace{\mathbf{x}_1, \mathbf{x}_2, \ldots}_{\text{spin-up}}, \underbrace{\mathbf{y}_1, \mathbf{y}_2, \ldots}_{\text{spin=down}})$$

When $|\mathbf{x}_i - \mathbf{y}_j| \to 0$:

$$\Psi \to \frac{C}{|\mathbf{x}_i - \mathbf{y}_j|} + 0 \times |\mathbf{x}_i - \mathbf{y}_j|^0 + O(|\mathbf{x}_i - y_j|)$$

Free gas corresponds to

$$\Psi \to \frac{0}{|\mathbf{x}_i - \mathbf{y}_j|} + C + O(|\mathbf{x}_i - y_j|)$$

# Field theory interpretation

Consider the following model

Sachdev, Nikolic; Nishida, DTS

$$S = \int dt\, d^d x \, \left( i\psi^\dagger \partial_t \psi - \frac{1}{2m} |\nabla \psi|^2 - c_0 \psi_\uparrow^\dagger \psi_\downarrow^\dagger \psi_\downarrow \psi_\uparrow \right)$$

Dimensional analysis:

$$[t] = -2, \quad [x] = -1, \quad [\psi] = \frac{d}{2}, \quad [c_0] = 2 - d$$

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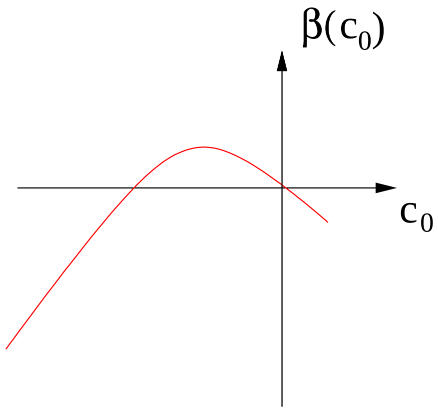
Contact interaction is irrelevant at $d > 2$
RG equation in $d = 2 + \epsilon$:

$$\frac{\partial c_0}{\partial s} = -\epsilon c_0 - \frac{c_0^2}{2\pi}$$

Two fixed points:

- $c_0 = 0$: trivial, noninteracting
- $c_0 = -2\pi\epsilon$: unitarity regime



# Field theory in $d = 4 - \epsilon$ dimensions

Sachdev, Nikolic; Nishida, DTS; Nussinov and Nussinov

$$S = \int dt\, d^d x \left( i\psi^\dagger \partial_t \psi - \frac{1}{2m} |\nabla \psi|^2 - g\phi \psi_\uparrow^\dagger \psi_\downarrow^\dagger - g\phi^* \psi_\downarrow \psi_\uparrow \right.$$

$$\left. + i\phi^* \partial_t \phi - \frac{1}{4m} |\nabla \phi|^2 + C\phi^* \phi \right)$$

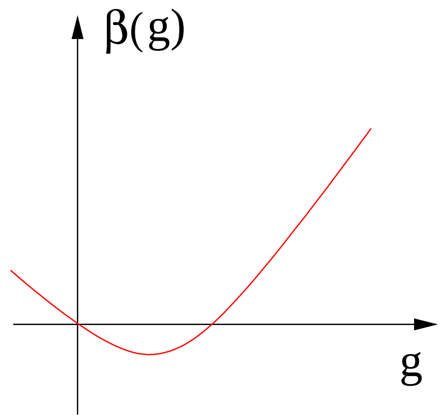
$C$ fined tuned to criticality

Dimensions: $[g] = \frac{1}{2}(4-d) = \frac{1}{2}\epsilon$

RG equation for $g$:

$$\frac{\partial g}{\partial \ln \mu} = -\frac{\epsilon}{2} g + \frac{g^3}{16\pi^2}$$

Fixed point at $g^2 = 8\pi^2 \epsilon$



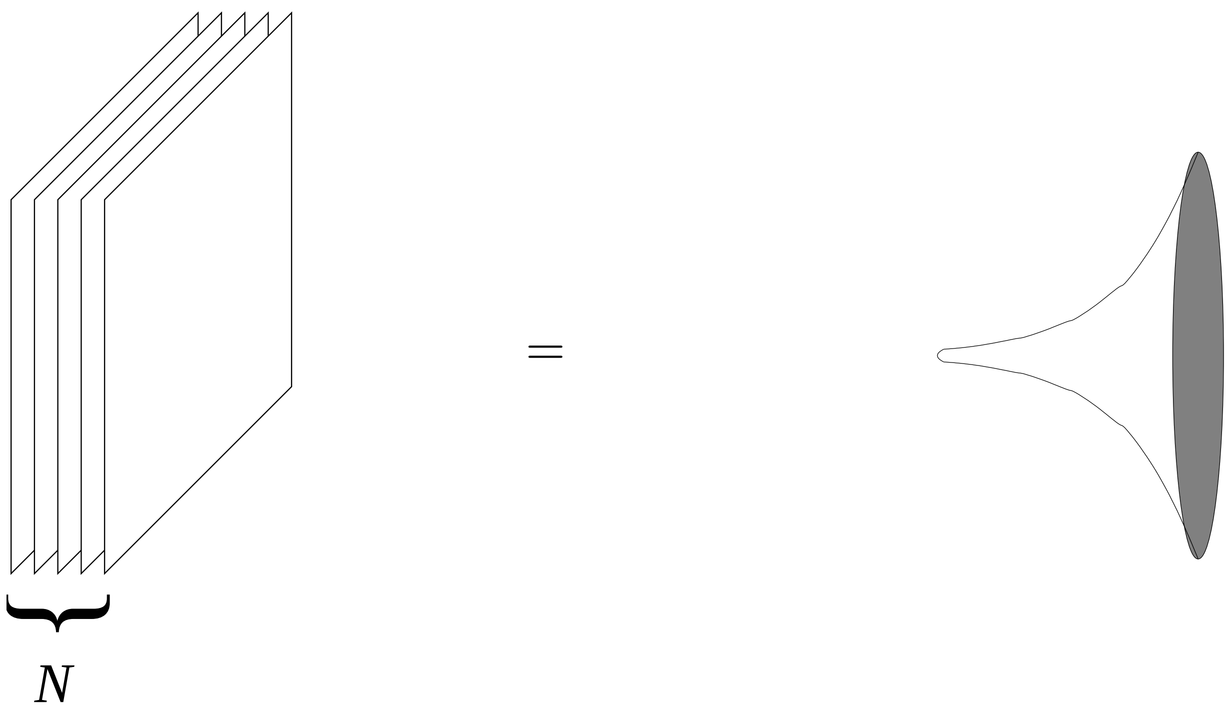
# AdS/CFT

Maldacena : stack of $N$ D3-branes in type IIB string theory can be described in two different pictures:

As a quantum field theory describing fluctuations of the branes: $\mathcal{N} = 4$ super-Yang-Mills theory

As string theory on a a curved spacetime called AdS$_5 \times$S$_5$

$$ds^2 = \frac{r^2}{R^2}(-dt^2 + d\mathbf{x}^2) + \frac{R^2}{r^2}dr^2 + R^2 d\Omega_5^2$$



# Applied string theory

AdS/CFT $\rightarrow$ solution of $\mathcal{N}=4$ SYM theory (and its cousins) at large $N$, large ’t Hooft coupling $\lambda$ limit

- Free energy at strong coupling

$$P(T)|_{\lambda\to\infty} = \frac{3}{4}P(T)_{\lambda\to 0}$$

- Shear viscosity: computable without kinetic theory

$$\frac{\eta}{s} = \frac{\hbar}{4\pi}$$

- Suggested modification of relativistic hydrodynamics in the presence of triangle anomalies:

$$j^\mu = \rho u^\mu + \text{diffusion} + \xi\epsilon^{\mu\nu\alpha\beta}u_\nu\partial_\alpha u_\beta$$

  Chiral separation in rotating relativistic fluids

# Analogy with AdS/CFT

Bertsch parameter:

$$\epsilon(n) = \xi \epsilon_{\text{free}}(n)$$

Current estimate: $\xi \approx 0.4$

Similar to pressure in $\mathcal{N} = 4$ SYM theory

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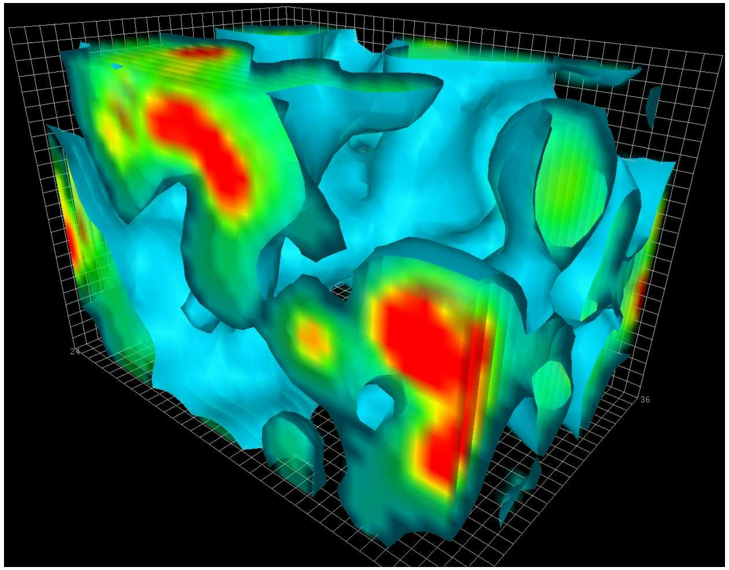
Is there an useful AdS/CFT-type duality for unitarity Fermi gas?

As in $\mathcal{N} = 4$ SYM, perhaps we should start with the vacuum (zero temperture and density)

- More symmetries
- Temperature and chemical potential can (hopefully) be accomodated later

# Vacuum

Relativistic vacuum



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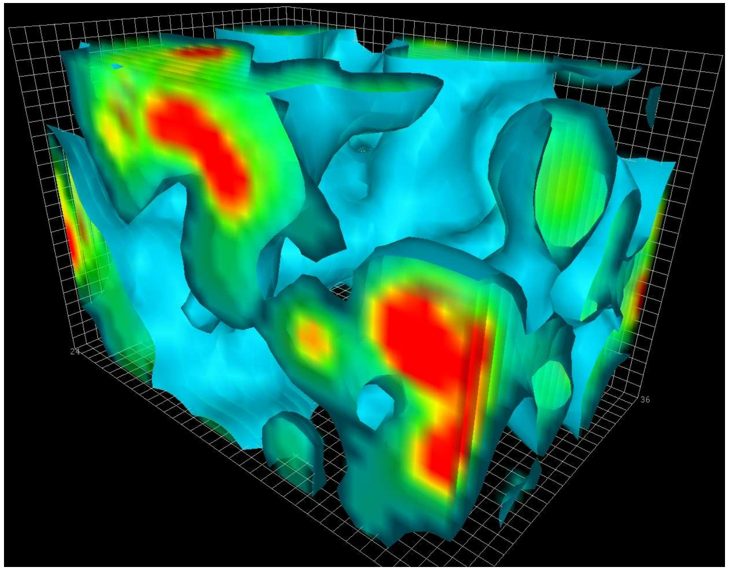

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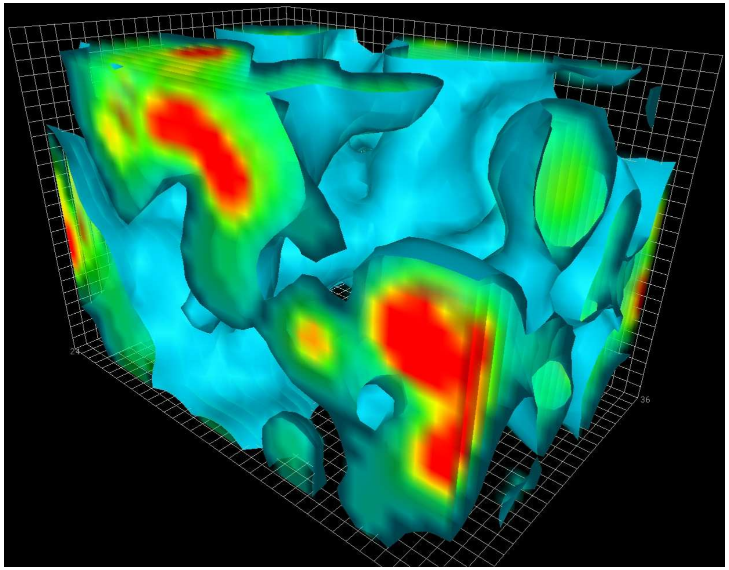


Nonrelativistic vacuum

Nonrelativistic vacuum is much simpler (no particle-hole pair creation)

Still: nontrivial conformal dimensions and correlation functions.

# AdS/CFT correspondence

$\mathcal{N} = 4$ super-Yang-Mills theory $\Leftrightarrow$ type IIB string theory on AdS$_5 \times$ S$^5$.

First evidence: matching of symmetries SO$(4,2) \times$ SO$(6)$.

- SO$(4,2)$: conformal symetry of 4-dim theories (CFT$_4$), isometry of AdS$_5$
- SO$(6)$: $\sim$ SU$(4)$ is the R-symmetry of $N = 4$ SYM, isometry of S$^5$.

Conformal algebra: $P^\mu$, $M^{\mu\nu}$, $K^\mu$, $D$

$$[D,\, P^\mu] = -iP^\mu, \qquad [D,\, K^\mu] = iK^\mu$$

$$[P^\mu,\, K^\nu] = -2i(g^{\mu\nu}D + M^{\mu\nu})$$

# Schrödinger algebra

Nonrelativistic field theory is invariant under:

- Phase rotation $M$: $\psi \to \psi e^{i\alpha}$
- Time and space translations, $H$ and $P^i$
- Rotations $M^{ij}$
- Galilean boosts $K^i$
- Dilatation $D$: $\mathbf{x} \to \lambda \mathbf{x}$, $t \to \lambda^2 t$
- Conformal transformation $C$:

$$\mathbf{x} \to \frac{\mathbf{x}}{1-\lambda t}, \qquad t \to \frac{t}{1-\lambda t}, \qquad \psi \to \cdots$$

$$[D,\, P^i] = -iP^i, \qquad [D,\, K^i] = iK^i, \qquad [P^i,\, K^j] = -\delta^{ij} M$$

$$[D,\, H] = -2iH, \qquad [D,\, C] = 2iC, \qquad [H,\, C] = iD$$

$D$, $H$, $C$ form a SO(2,1)

Chemical potential $\mu\psi^\dagger\psi$ breaks the symmetry.

# Generators

$$M = \int d\mathbf{x}\, n(\mathbf{x}), \qquad P_i = \int d\mathbf{x}\, j_i(\mathbf{x})$$

$$K_i = \int d\mathbf{x}\, x_i n(\mathbf{x}), \qquad C = \frac{1}{2}\int d\mathbf{x}\, x^2 n(x), \qquad D = -\int d\mathbf{x}\, x_i j_i(\mathbf{x})$$

$H \to H + \omega^2 C$: putting the system in an external potential $V(\mathbf{x}) = \frac{1}{2}\omega^2 x^2$.

Operator $\mathcal{O}$ has dimension $\Delta$ if $[D,\, \mathcal{O}(0)] = -i\Delta\mathcal{O}(0)$

$[D,\, K_i] = iK_i$: $K_i$ lowers dimension by 1

$[D,\, C] = 2iC$: $C$ lowers dimension by 2

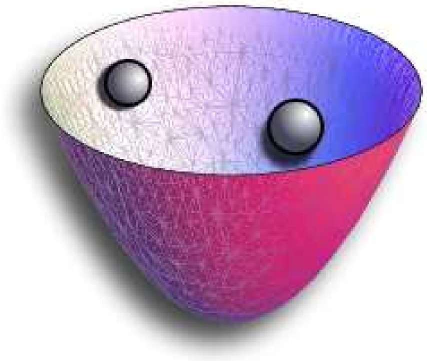
Define primary operators: $[K_i,\, \mathcal{O}(0)] = [C,\, \mathcal{O}(0)] = 0$

Examples of primary operator for free fermions:
$\psi_\uparrow\psi_\downarrow$, $\psi_\uparrow\partial_i\psi_\downarrow - \partial_i\psi_\uparrow\psi_\downarrow$, but not $\partial_i(\psi_\uparrow\psi_\downarrow)$

# Operator-state correspondence

Nishida, DTS

Primary operator with dimension $\Delta$ $\Longleftrightarrow$ eigenstate in harmonic potential with energy $\Delta \times \hbar\omega$

$\psi_\uparrow \psi_\downarrow$ $\Longleftrightarrow$ 

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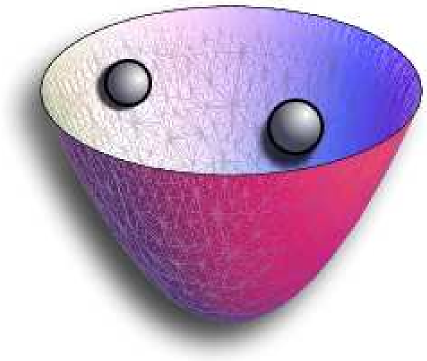
$\psi_\uparrow \psi_\downarrow$ $\Longleftrightarrow$ 

cf. Werner, Castin; Tan

Proof: let $\omega = 1$, $H_{\rm osc} = H + C$; Define $|\Psi_{\mathcal{O}}\rangle = e^{-H} \mathcal{O}^\dagger(0)|0\rangle$

From Schrödinger algebra one finds $e^H H_{\rm osc} e^{-H} = C + iD$

from $[C, \mathcal{O}^\dagger(0)] = 0$, $[D, \mathcal{O}^\dagger(0)] = -i\Delta_{\mathcal{O}}$, and $C|0\rangle = D|0\rangle = 0$:

$$H_{\rm osc}|\Psi_{\mathcal{O}}\rangle = \Delta_{\mathcal{O}}|\Psi_{\mathcal{O}}\rangle$$

# Example

Two particles in harmonic potential: ground state with unitarity boundary condition can be found exactly

$$\Psi(\mathbf{x},\mathbf{y}) = \frac{e^{-(x^2+y^2)/2}}{|\mathbf{x}-\mathbf{y}|}\,, \qquad E_0 = 2\hbar\omega$$

$\longrightarrow$ Dimension of operator $O_2 = \psi_\uparrow\psi_\downarrow$ is 2 (naively 3)

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Can be see explicitely: two-particle wavefunctions behave as

$$\Psi(\mathbf{x},\mathbf{y}) \sim \frac{1}{|\mathbf{x}-\mathbf{y}|}, \qquad x \to y$$

the operator $O_2$ has to be regularized as

$$O_2(\mathbf{x}) = \lim_{\mathbf{x}\to\mathbf{y}} |\mathbf{x}-\mathbf{y}| \psi_\uparrow(\mathbf{x}) \psi_\downarrow(\mathbf{y})$$

so that $\langle 0 | O_2 | \Psi(\mathbf{x},\mathbf{y}) \rangle$ is finite.

Lowest 3-body operators: $\Delta_{l=1} = 4.27272...$, $\Delta_{l=0} = 4.66622...$

# Relation to Shina Tan's contanct

Shina Tan has derived several exact relations, for example between total energy and occupation number:

$$E = \sum_\sigma \int \frac{d\mathbf{p}}{(2\pi)^3} \frac{p^2}{2m} \left( n_{\mathbf{p},\sigma} - \frac{C}{p^4} \right) + \frac{V}{4\pi a m} C$$

The connection between $C$ and operator $O_2$ is

$$C = \langle O_2^\dagger(\mathbf{x}) O_2(\mathbf{x}) \rangle$$

# Embedding the Schrödinger algebra

- Sch$(d)$: is the symmetry of the Schrödinger equation

$$i\frac{\partial\psi}{\partial t}+\frac{\nabla^2}{2m}\psi=0$$

- CFT$_{d+2}$: is the symmetry of the Klein-Gordon equation

$$\partial_\mu^2\phi=0, \qquad \mu=0,1,\cdots,d+1$$

In light-cone coordinates $x^\pm=x^0\pm x^{d+1}$ the Klein-Gordon equation becomes

$$-2\frac{\partial}{\partial x^+}\frac{\partial}{\partial x^-}\phi+\partial_i\partial_i\phi=0, \qquad i=1,\cdots,d$$

Restricting $\phi$ to $\phi=e^{-imx^-}\psi(x^+,\mathbf{x})$: Klein-Gordon eq. $\Rightarrow$ Schrödinger eq.:

$$2im\frac{\partial}{\partial x^+}\psi+\nabla^2\psi=0, \qquad \nabla^2=\sum_{i=1}^{d}\partial_i^2$$

This means Sch$(d)\subset$ CFT$_{d+2}$

# Embedding (2)

Sch$(d)$ is the subgroup of CFT$_{d+2}$ containing group elements which does not change the ansatz

$$\phi = e^{imx^-}\psi(x^+, x^i)$$

Algebra: sch$(d)$ is the subalgebra of the conformal algebra, containing elements that commute with $P^+$

$$[P^+, O] = 0$$

One can identify the Schrödinger generators:

$$M = P^+, \quad H = P^-, \quad K^i = M^{i-},$$

$$D_{\rm nonrel} = D_{\rm rel} + M^{+-}, \quad C = \frac{1}{2}K^+$$

# Nonrelativistic dilatation

$$\begin{array}{ccccc} & D_{\rm rel} & & M^{+-} & \\ x^+ & \longrightarrow & \lambda x^+ & \longrightarrow & \lambda^2 x^+ \\ x^- & \longrightarrow & \lambda x^- & \longrightarrow & x^- \\ x^i & \longrightarrow & \lambda x^i & \longrightarrow & \lambda x^i \end{array}$$

# Geometric realization of Schrödinger algebra

Start from $\mathrm{AdS}_{d+3}$ space:

$$ds^2 = \frac{-2dx^+dx^- + dx^i dx^i + dz^2}{z^2}$$

Invariant under the whole conformal group, in particular with respect to relativistic scaling

$$x^\mu \to \lambda x^\mu, \qquad z \to \lambda z$$

and boost along the $x^{d+1}$ direction:

$$x^+ \to \tilde{\lambda} x^+, \qquad x^- \to \tilde{\lambda}^{-1} x^-$$

Break the symmetry down to $\mathrm{Sch}(d)$:

$$ds^2 = \frac{-2dx^+dx^- + dx^i dx^i + dz^2}{z^2} - \frac{2(dx^+)^2}{z^4}$$

The additional term is invariant only under a combination of relativistic dilation and boost:

$$x^+ \to \lambda^2 x^+, \quad x^- \to x^-, \quad x^i \to \lambda x^i, \qquad z \to \lambda z$$

# Model

$$ds^2 = \frac{-2dx^+dx^- + dx^i dx^i + dz^2}{z^2} - \frac{2(dx^+)^2}{z^4}$$

Is there a model where this is a solution to the Einstein equation?

The additional term gives rise to a change in $R_{++} \sim z^{-4}$: we need matter that provides $T_{++} \sim z^{-4}$.

Can be provided by $A_\mu$ with $A_+ \sim 1/z^2$: has to be a massive gauge field.

$$S = \int d^{d+3}x \sqrt{-g} \left( \frac{1}{2}R - \Lambda - \frac{1}{4}F_{\mu\nu}^2 - \frac{m^2}{2}A_\mu^2 \right)$$

# Two-point function

Following standard prescription

$$S = -\int d^{d+3}x\,\sqrt{-g}(g^{\mu\nu}\partial_\mu\phi^*\partial_\mu\phi + m_0^2\phi^*\phi)$$

Consider $\phi \sim e^{iMx^-}$

$$S = \int d^{d+2}x\,dz\,\frac{1}{z^{d+3}}\left(2iMz^2\phi^*\partial_t\phi - z^2\partial_i\phi^*\partial_i\phi - m^2\phi^*\phi\right), \quad m^2 = m_0^2 + 2M^2$$

$$\langle O(\tau,\mathbf{x})O(0,0)\rangle \sim \frac{\theta(\tau)}{\tau^\Delta}\exp\left(-\frac{Mx^2}{2\tau}\right)$$

where

$$\Delta = \frac{d+2}{2} + \nu, \qquad \nu = \sqrt{m^2 + \frac{(d+2)^2}{4}}$$

Form dictated by Schrödinger symmetry.

# Three-point function

$$\langle O_1(t_1,x_1)O_2(t_2,x_2)O_3^\dagger(0,0)\rangle = \frac{\theta(t_1)\theta(t_2)}{t_1^{\Delta_{13,2}}t_2^{\Delta_{23,1}}(t_1-t_2)^{\Delta_{12,3}}}\exp\left[-\frac{M_1x_1^2}{2t_1}-\frac{M_2x_2^2}{2t_2}\right]\Psi(y)$$

$\Delta_{ij,k}=\Delta_i+\Delta_j-\Delta_k$, where $y$ is a Schrödinger invariant

$$y=\frac{x_1t_2-x_2t_1}{(t_1-t_2)t_1t_2}$$

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$$\Psi(y)\sim y^{1-d/2}\gamma\left(\frac{d}{2}-1,y\right) \longleftarrow \text{incomplete beta function}$$

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Holography: computing Witten diagram Fuertes, Moroz

$$\psi(y) \sim \int dv\,dv'\,e^{-iM_1v-iM_2v'}(v-v'+iy)^{-\Delta_{12,3}/2}(v')^{-\Delta_{23,1}}v^{-\Delta_{13,2}}$$

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specializing for $M_1 = M_2 = 1$, $\Delta_1 = \Delta_2 = d/2$, $\Delta_3 = 2$:

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  One bulk theory corresponds to two boundary theories with $\Delta_\pm = \frac{d+2}{2} \pm \nu$, $\nu < 1$

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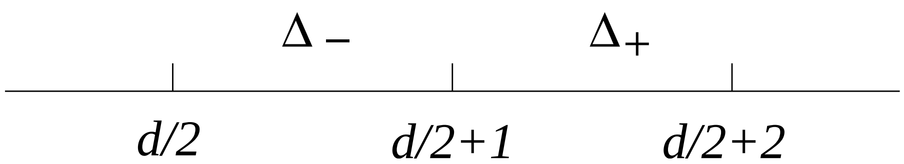
  One bulk theory corresponds to two boundary theories with $\Delta_\pm = \frac{d+2}{2} \pm \nu$, $\nu < 1$

- For a theory where some opeartor $O$ has dimension $\Delta[O] = \Delta_+$,
  $d/2 + 1 < \Delta_+ < d/2 + 2$
  there exist another theory where $O$ has dimension $\Delta_- = d + 2 - \Delta_+$,
  $d/2 < \Delta_- < d/2 + 1$



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- Free fermions: $[\psi_\uparrow \psi_\downarrow] = 3$

  3D: $\dfrac{d}{2} + 1 = 5/2$ $\qquad\qquad$ $\dfrac{d}{2} + 2 = 7/2$

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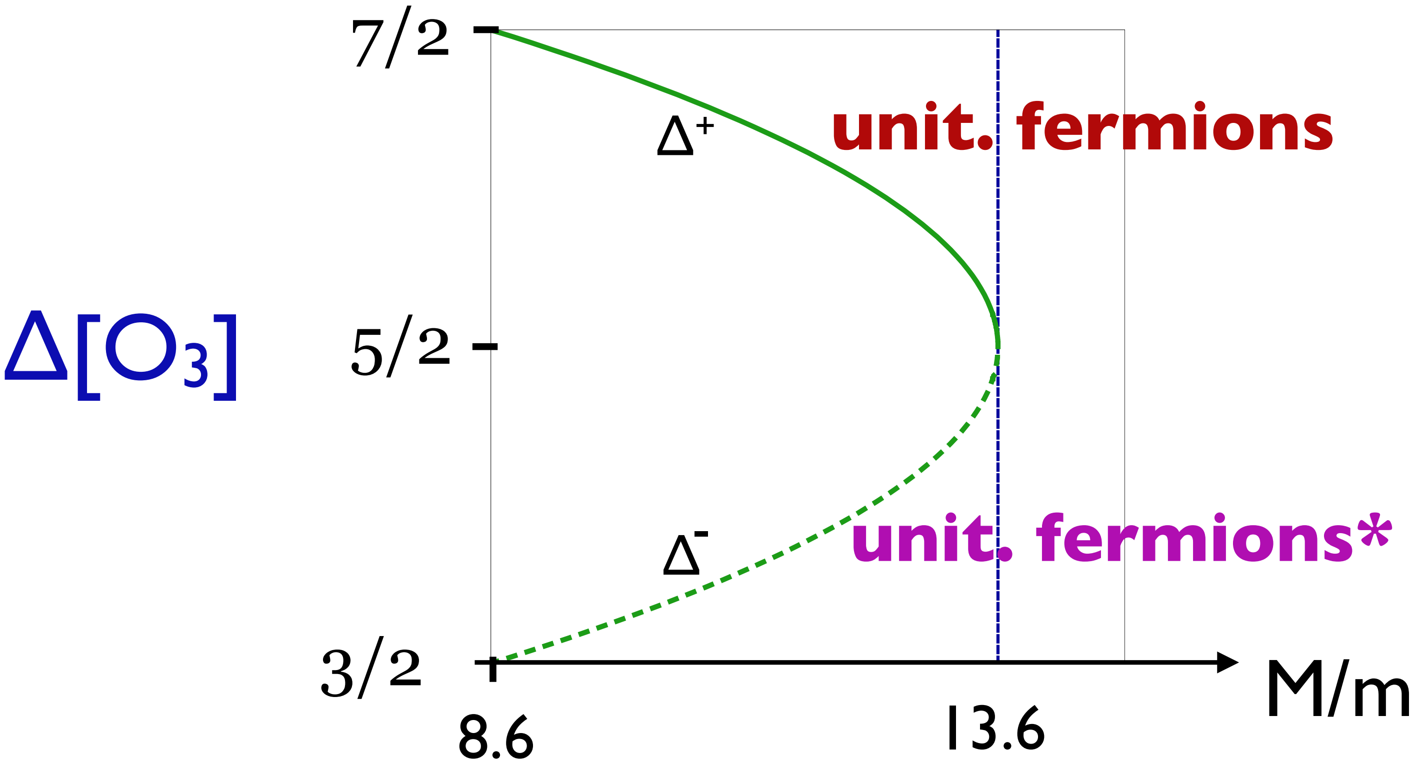
Example 3: one-dimensional unitarity fermions Nishida, Tan

- In 1d: $[\psi] = 1/2$
- $[\psi_\uparrow \psi_\downarrow] = 1$, outside $(3/2, 5/2)$
- 4 types of fermions: $[\psi_1 \psi_2 \psi_3 \psi_4] = 2$
- 1D system with contact 4-body interaction, fine-tuned to unitarity

# Examples (continued)

- Unitarity fermions with different masses for 2 flavors $M/m \neq 1$
- One three-body operator $\Psi\partial_i\Psi\psi$: dimension between $7/2$ and $5/2$ when $M/m$ varies from 8.6 to 13.6
- There exists another scale-invariant theory with two and three-body resonances in this range of mass ratio Nishida, DTS, Shina Tan

# Unitarity fermions*



# Black holes

Schrödinger metric Can be realized in string theory ($d = 2$)

Herzog, Rangamani, Ross;

Maldacena, Martelli, Tachikawa;

Adam, Balasubramanian, McGreevy (2008)

$$S = \int d^5x \sqrt{-g} \left( \frac{1}{2}R - \Lambda - \frac{1}{4}F_{\mu\nu}^2 - \frac{m^2}{2}A_\mu^2 \right) + \text{scalars}$$

Black-hole solutions also constructed: allow studying hydrodynamics

- Shear viscosity: $\eta/s = 1/(4\pi)$
- Thermal conductivity: Prandtl number $= 1$
- Thermodynamics $P(T, \mu) = \#T^4/\mu^2$: unrealistic

Why such equation of state?

- Dimensional reduction on light cone leaves an infinite number of fields with growing particle number (mass)
- They all contribute to the free energy Barbon, Fuertes

# What we don't know

- Holographic renormalization
- Role of large $N$? (Sp($2N$) model?)
- Hierarchical organization in NR field theories
  - to understand $n$-body sector we don't need to know solution to $n+1$, $n+2$ etc. body problem
  - In gravity: throwing away fields with mass $> n$ should be a consistent truncation
  - Not a feature of current string constructions of Schrödinger background
- Maybe one should get rid of the light-cone construction and start anew using a nonrelativistic gravity from the start (Newton-Cartan gravity?)

# Speculations

More generally, AdS/CFT suggests that one can think about quantum field theories in a completely different language

- Perhaps one can reformulate unitarity Fermi gas in the language of local operators and their OPE cf. Braaten, Platter
- Infinite number of operators, corresponding to short-range behavior of wavefunctions/ eigenstates in harmonic potential
- A Schwinger-Dyson-type hierarchy?
- Truncation to $\psi$ and $O_2$ (+ additional approximations) $\Rightarrow$ BCS mean-field theory?
- Keeping more fields $\rightarrow$ more accurate approximations?

# Conclusion

- Unitarity fermions have Schrödinger symmetry, a nonrelativistic conformal symmetry
- Universal properties, studied in experiments
- There is a metric with Schrödinger symmetry
- Starting point for dual phenomenology of unitarity fermions?
- Deeper connection between few- and many-body physics?