

# In Search of the Perfect Fluid

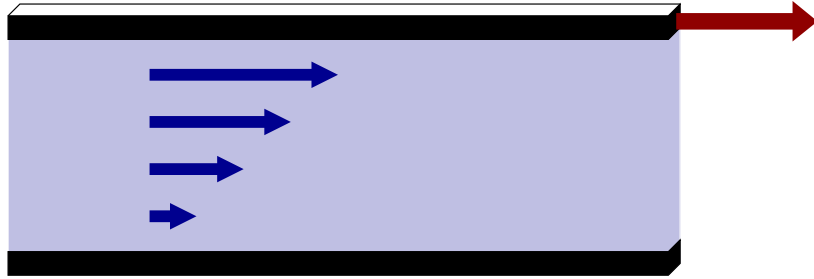
Thomas Schaefer, North Carolina State University



See T. Schäfer, D. Teaney, “Perfect Fluidity” [arXiv:0904.3107]

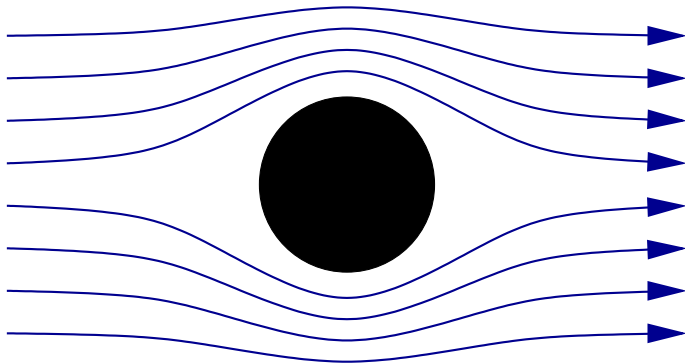
# Measures of Perfection

Viscosity determines shear stress (“friction”) in fluid flow



$$F = A \eta \frac{\partial v_x}{\partial y}$$

Dimensionless measure of shear stress: Reynolds number



$$Re = \frac{n}{\eta} \times mvr$$

fluid property                      flow property

- $[\eta/n] = \hbar$

- Relativistic systems  $Re = \frac{s}{\eta} \times \tau T$

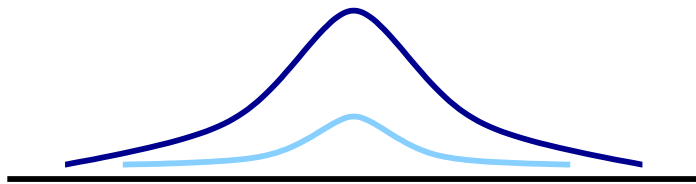
Other sources of dissipation (thermal conductivity, bulk viscosity, ...) vanish for certain fluids, but shear viscosity is always non-zero.

There are reasons to believe that  $\eta$  is bounded from below by a constant times  $\hbar s/k_B$ . In a large class of theories  $\eta/s \geq \hbar/(4\pi k_B)$ .

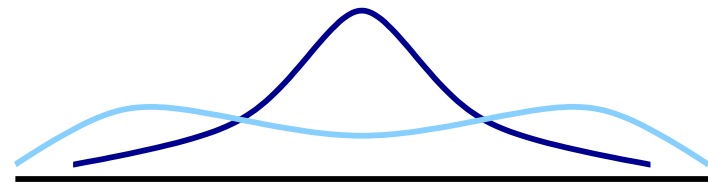
A fluid that saturates the bound is a “perfect fluid”.

# Fluids: Gases, Liquids, Plasmas, ...

Hydrodynamics: Long-wavelength, low-frequency dynamics of conserved or spontaneously broken symmetry variables.



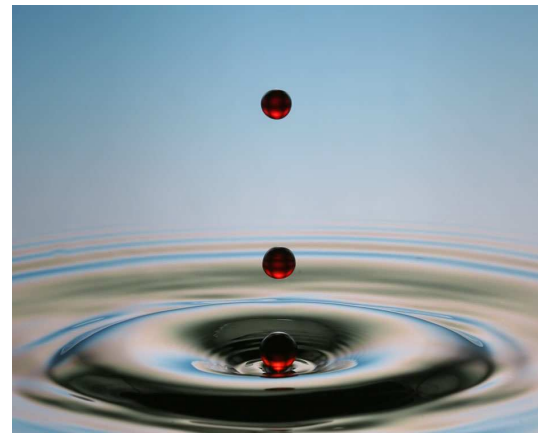
$$\tau \sim \tau_{micro}$$



$$\tau \sim \lambda^{-1}$$

Historically: Water

$$(\rho, \epsilon, \vec{\pi})$$



## Example: Simple Fluid

Conservation laws: mass, energy, momentum

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0$$

$$\frac{\partial \epsilon}{\partial t} + \vec{\nabla}_j \vec{j}^\epsilon = 0$$

$$\frac{\partial}{\partial t} (\rho v_i) + \frac{\partial}{\partial x_j} \Pi_{ij} = 0$$

[Euler/Navier-Stokes equation]



Constitutive relations: Energy momentum tensor

$$\Pi_{ij} = P\delta_{ij} + \rho v_i v_j + \eta \left( \partial_i v_j + \partial_j v_i - \frac{2}{3} \delta_{ij} \partial_k v_k \right) + O(\partial^2)$$

reactive

dissipative

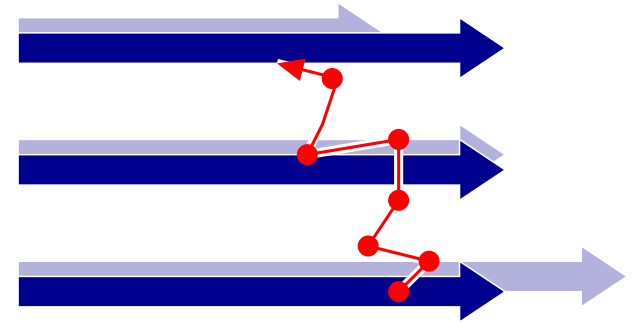
2nd order

# Kinetic Theory

Kinetic theory: conserved quantities carried by quasi-particles

$$\frac{\partial f_p}{\partial t} + \vec{v} \cdot \vec{\nabla}_x f_p + \vec{F} \cdot \vec{\nabla}_p f_p = C[f_p]$$

$$\eta \sim \frac{1}{3} n \bar{p} l_{mfp}$$



Normalize to density. Uncertainty relation suggests

$$\frac{\eta}{n} \sim \bar{p} l_{mfp} \geq \hbar$$

Also:  $s \sim k_B n$  and  $\eta/s \geq \hbar/k_B$

Validity of kinetic theory as  $\bar{p} l_{mfp} \sim \hbar?$

# Effective Theories for Fluids (Here: Weak Coupling QCD)



$$\mathcal{L} = \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4} G_{\mu\nu}^a G_{\mu\nu}^a$$



$$\frac{\partial f_p}{\partial t} + \vec{v} \cdot \vec{\nabla}_x f_p = C[f_p] \quad (\omega < T)$$



$$\frac{\partial}{\partial t} (\rho v_i) + \frac{\partial}{\partial x_j} \Pi_{ij} = 0 \quad (\omega < g^4 T)$$

# Holographic Duals: Transport Properties

Thermal (conformal) field theory  $\equiv AdS_5$  black hole

CFT entropy  $\Leftrightarrow$

Hawking-Bekenstein entropy

$\sim$  area of event horizon

shear viscosity  $\Leftrightarrow$

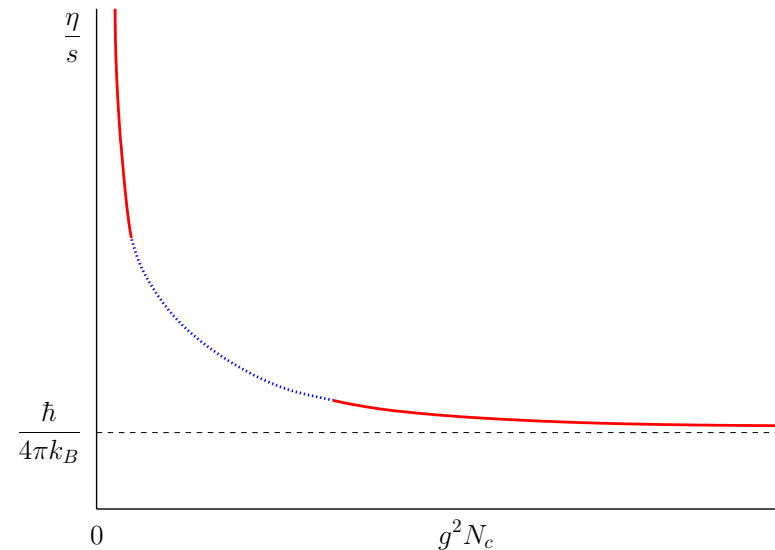
Graviton absorption cross section

$\sim$  area of event horizon

Strong coupling limit

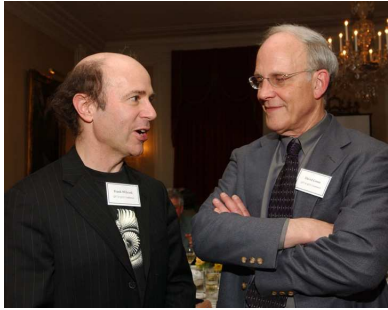
$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

Son and Starinets (2001)



Strong coupling limit universal? Provides lower bound for all theories?

# Effective Theories (Strong coupling)



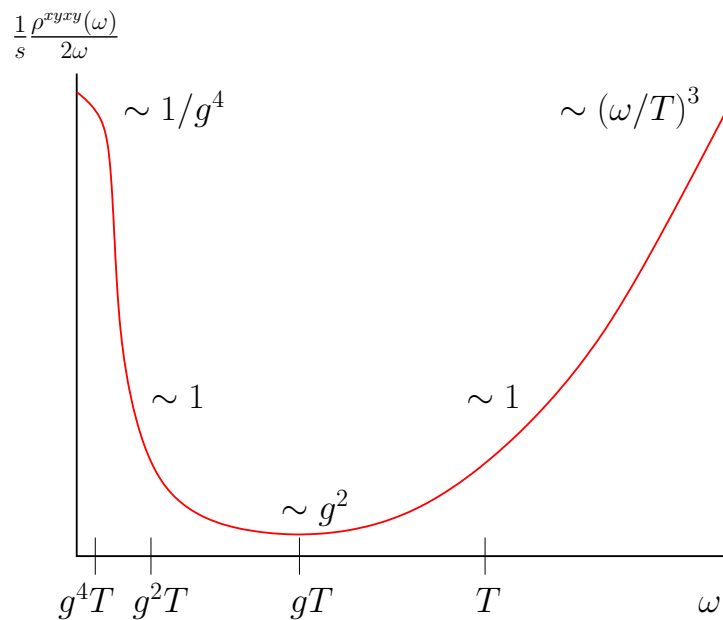
$$\mathcal{L} = \bar{\lambda}(i\sigma \cdot D)\lambda - \frac{1}{4}G_{\mu\nu}^a G_{\mu\nu}^a + \dots \Leftrightarrow S = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g} \mathcal{R} + \dots$$



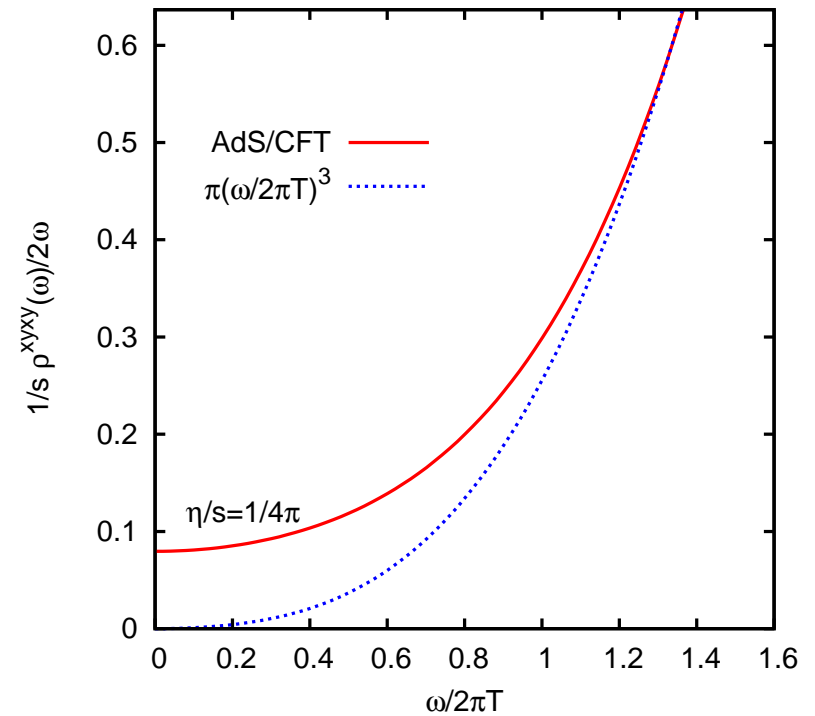
$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j} \Pi_{ij} = 0 \quad (\omega < T)$$

# Kinetics vs No-Kinetics

Spectral function  $\rho(\omega) = \text{Im}G_R(\omega, 0)$  associated with  $T_{xy}$



weak coupling QCD



strong coupling AdS/CFT

transport peak vs no transport peak

## Perfect Fluids: How to be a contender?

Bound is quantum mechanical

need quantum fluids

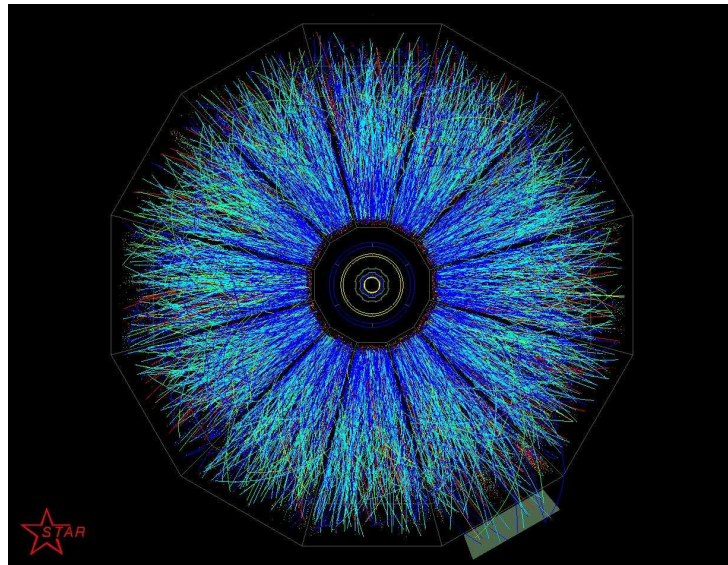
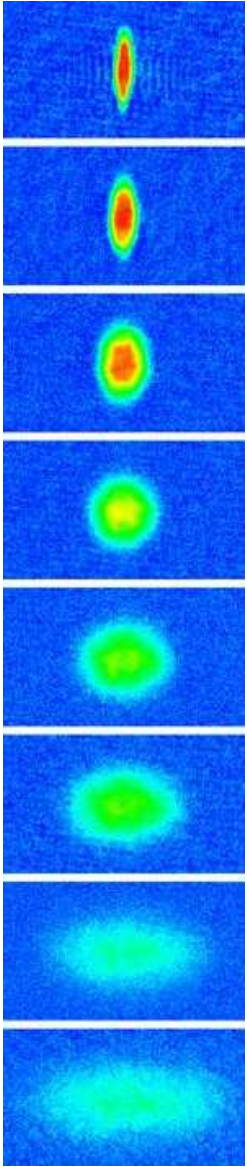
Bound is incompatible with weak coupling and kinetic theory

strong interactions, no quasi-particles

Model system has conformal invariance (essential?)

(Almost) scale invariant systems

# Perfect Fluids: The contenders



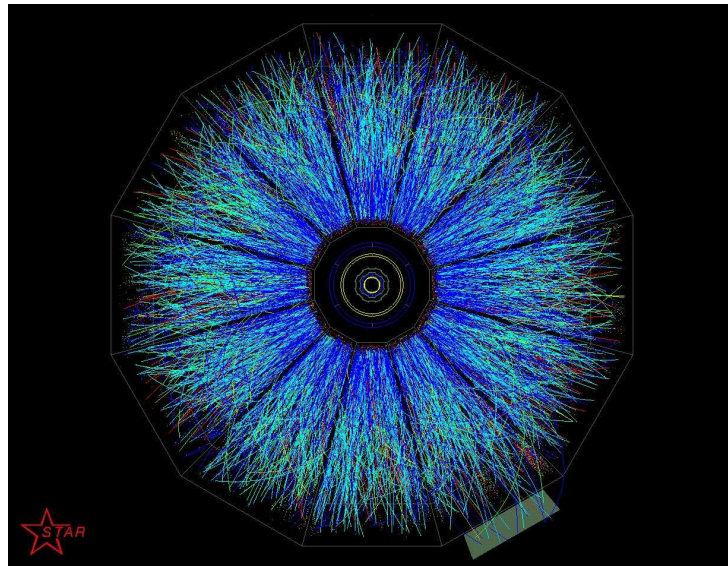
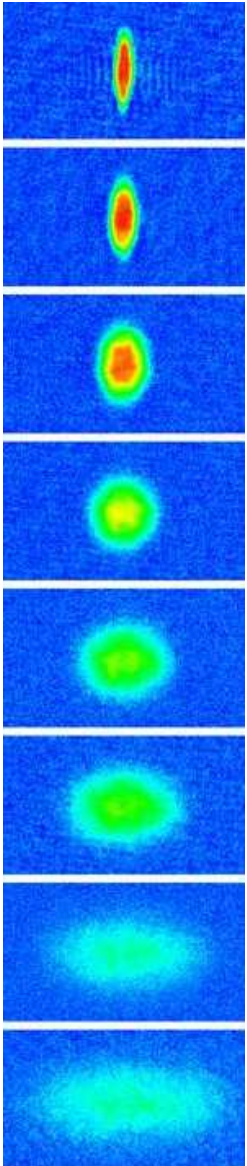
QGP ( $T=180$  MeV)

Trapped Atoms  
( $T=0.1$  neV)



Liquid Helium  
( $T=0.1$  meV)

# Perfect Fluids: The contenders



QGP  $\eta = 5 \cdot 10^{11} Pa \cdot s$

Trapped Atoms

$\eta = 1.7 \cdot 10^{-15} Pa \cdot s$



Liquid Helium

$\eta = 1.7 \cdot 10^{-6} Pa \cdot s$

Consider ratios

$\eta/s$

# Kinetic Theory: Quasiparticles

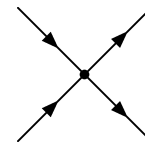
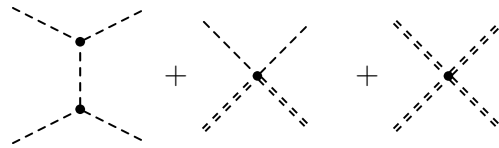
low temperature

high temperature

helium

phonons, rotons

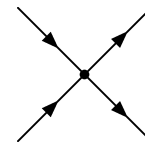
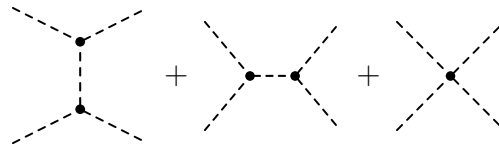
atoms



unitary gas

phonons

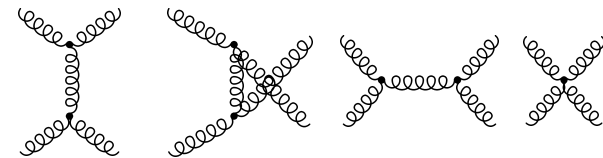
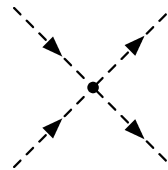
atoms



QCD

pions

quarks, gluons



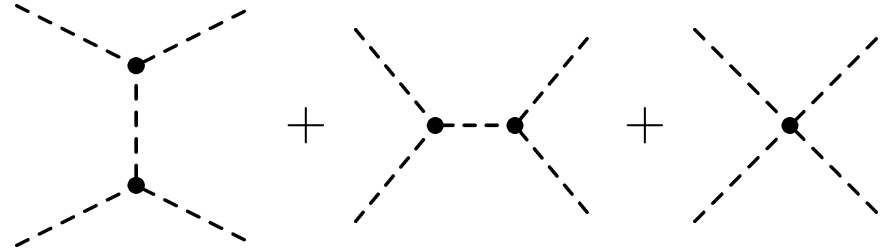
Low T: Phonons Goldstone boson  $\psi\psi = e^{2i\varphi} \langle \psi\psi \rangle$

$$\mathcal{L} = c_0 m^{3/2} \left( \mu - \dot{\varphi} - \frac{(\vec{\nabla}\varphi)^2}{2m} \right)^{5/2} + \dots$$

Viscosity dominated by  $\varphi + \varphi \rightarrow \varphi + \varphi$

$$\eta = A \frac{\xi^5}{c_s^3} \frac{T_F^8}{T^5}$$

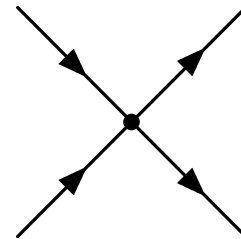
T.S., G.R. (2007)



High T: Atoms Cross section regularized by thermal momentum

$$\eta = \frac{15}{32\sqrt{2}} (mT)^{3/2}$$

Bruun (2005)



## Low T: Phonons and Rotons Effective Lagrangian

$$\begin{aligned}\mathcal{L} = & \varphi^* (\partial_0^2 - v^2) \varphi + i\lambda \dot{\varphi} (\vec{\nabla} \varphi)^2 + \dots \\ & + \varphi_{R,v}^* (i\partial_0 - \Delta) \varphi_{R,v} + c_0 (\varphi_{R,v}^* \varphi_{R,v})^2 + \dots\end{aligned}$$

Shear viscosity

$$\eta = \eta_R + \frac{c_{Rph}}{\sqrt{T}} e^{\Delta/T} + \frac{c_{ph}}{T^5} + \dots$$

Landau & Khalatnikov

High T: Atoms Viscosity governed by hard core ( $V \sim 1/r^{12}$ )

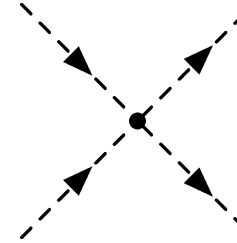
$$\eta = \eta_0 (T/T_0)^{2/3}$$

Low T: Pions Chiral perturbation theory

$$\mathcal{L} = \frac{f_\pi^2}{4} \text{Tr}[\partial_\mu U \partial^\mu U^\dagger] + (B \text{Tr}[MU] + h.c.) + \dots$$

Viscosity dominated by  $\pi\pi$  scattering

$$\eta = A \frac{f_\pi^4}{T}$$



High T: Quasi-Particles HTL theory (screening, damping, ...)

$$\mathcal{L}_{HTL} = \int d\Omega G_{\mu\alpha}^a \frac{v^\alpha v_\beta}{(v \cdot D)^2} G^{a,\mu\beta}$$

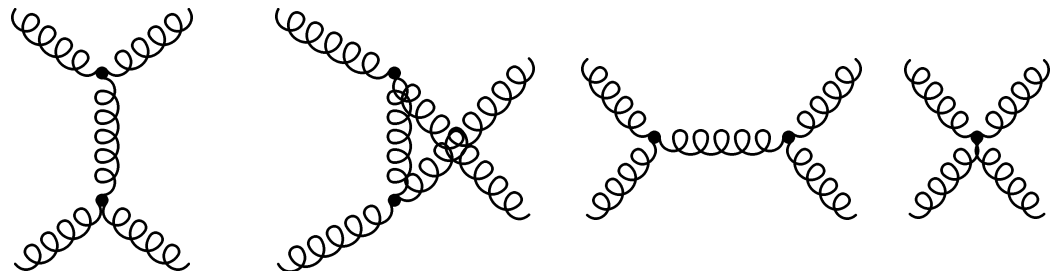
quasi-particle width

$$\gamma \sim g^2 T$$

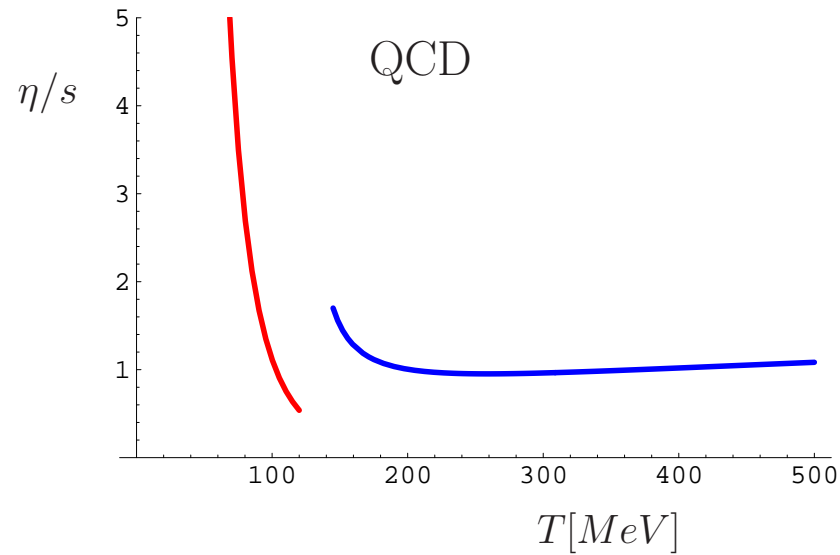
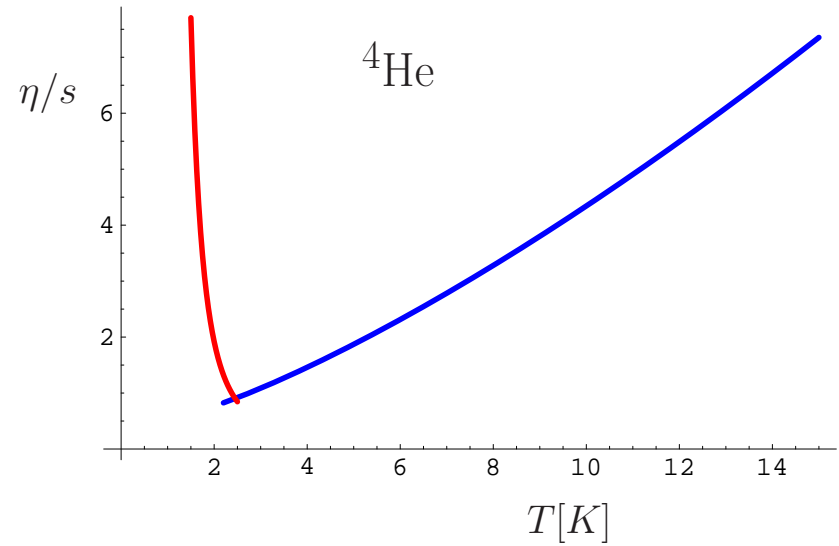
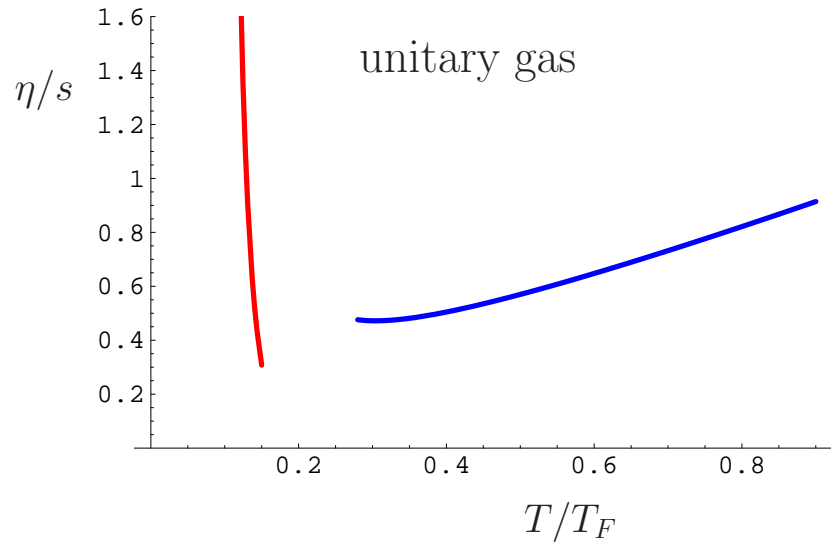
Viscosity dominated by t-channel gluon exchange

$$\eta = \frac{27.13 T^3}{g^4 \log(2.7/g)}$$

AMY (2003)



# Theory Summary



# I. Experiment (Liquid Helium)

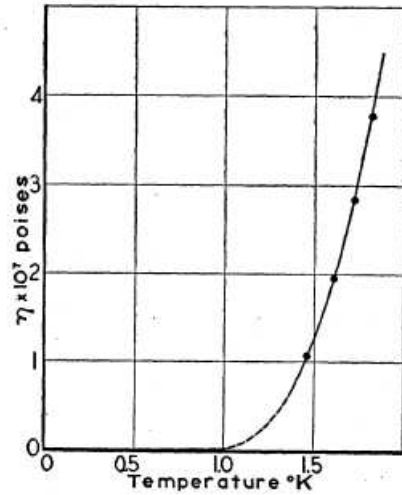
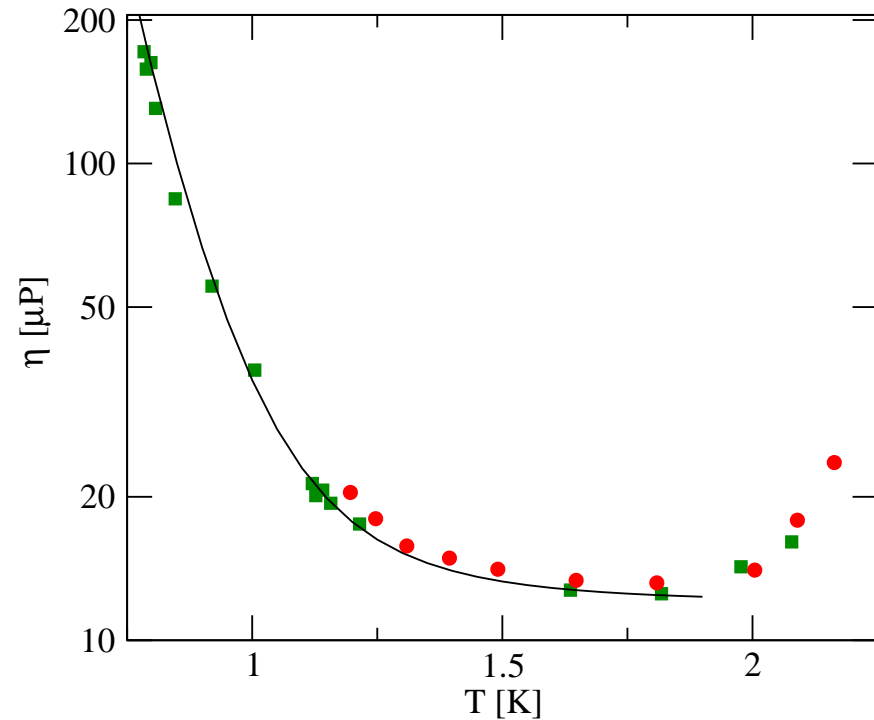


FIG. 1. The viscosity of liquid helium II measured by flow through a  $10^{-4}$  cm channel.



Kapitza (1938)

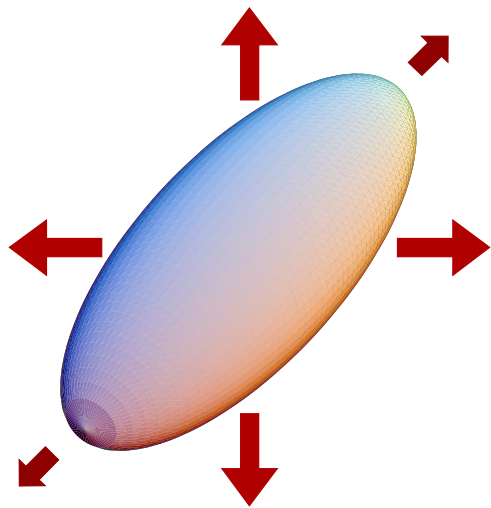
viscosity vanishes below  $T_c$   
capillary flow viscometer

Hollis-Hallett (1955)

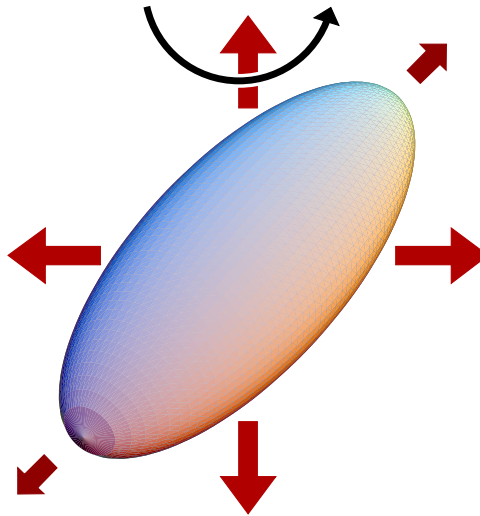
roton minimum, phonon rise  
rotation viscometer

$$\eta/s \simeq 0.8 \hbar/k_B$$

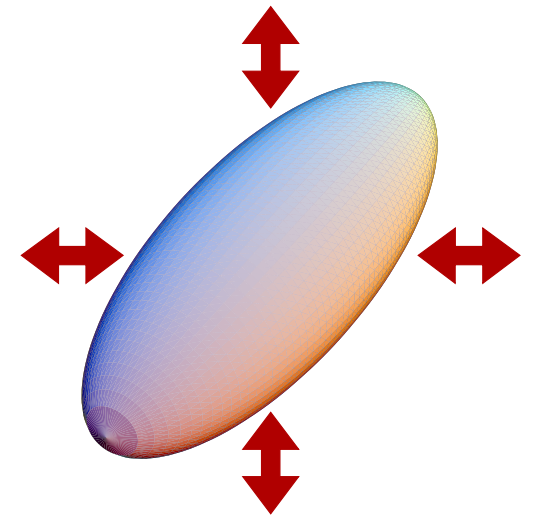
## II. Scaling Flows (Cold Gases)



transverse expansion



expansion (rotating trap)



collective modes

$$\frac{\partial n}{\partial t} + \vec{\nabla} \cdot (n\vec{v}) = 0$$
$$mn \frac{\partial \vec{v}}{\partial t} + mn (\vec{v} \cdot \vec{\nabla}) \vec{v} = -\vec{\nabla} P - n \vec{\nabla} V$$

# Scaling Flows

Universal equation of state

$$P = \frac{n^{5/3}}{m} f\left(\frac{mT}{n^{2/3}}\right)$$

Equilibrium density profile

$$n_0(x) = n(\mu(x), T) \quad \mu(x) = \mu_0 \left(1 - \frac{x^2}{R_x^2} - \frac{y^2}{R_y^2} - \frac{z^2}{R_z^2}\right)$$

Scaling Flow: Stretch and rotate profile

$$\mu_0 \rightarrow \mu_0(t), \quad T \rightarrow T_0(\mu_0(t)/\mu_0), \quad R_x \rightarrow R_x(t), \quad \dots$$

Linear velocity profile

$$\vec{v}(x, t) = (\alpha_x x + (\alpha - \omega)y, \alpha_y y + (\alpha + \omega)y, \alpha_z z)$$

“Hubble flow”

## Dissipation (Scaling Flows)

Energy dissipation ( $\eta, \zeta, \kappa$ : shear, bulk viscosity, heat conductivity)

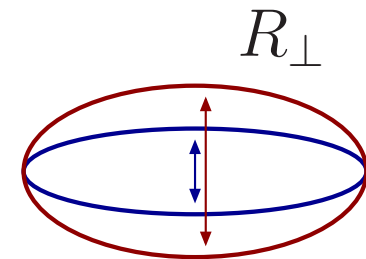
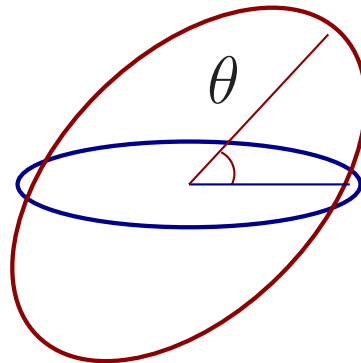
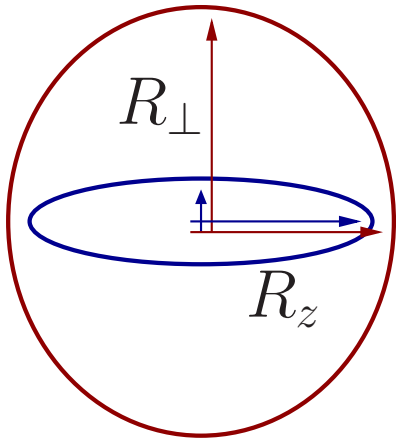
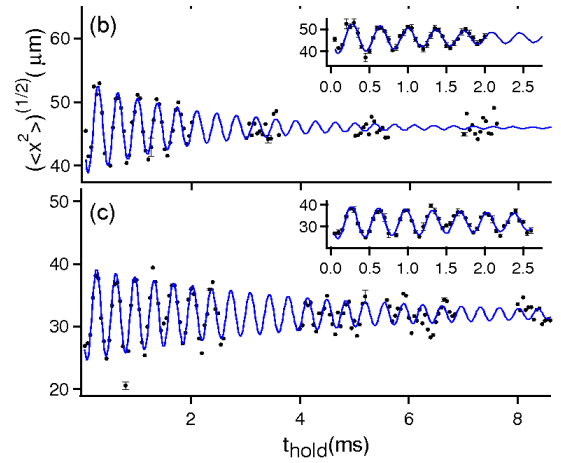
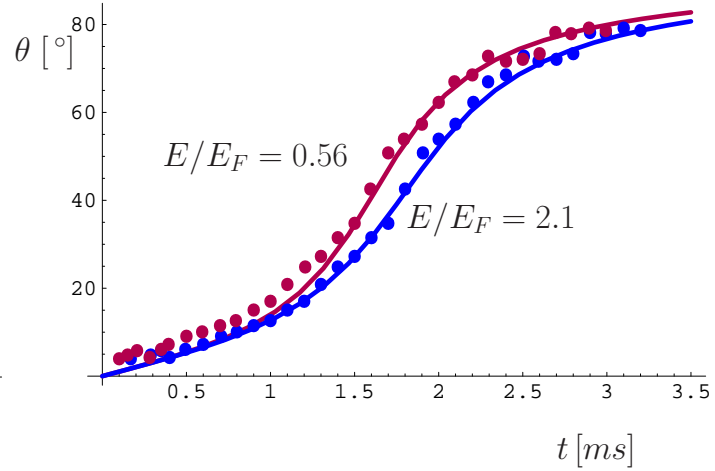
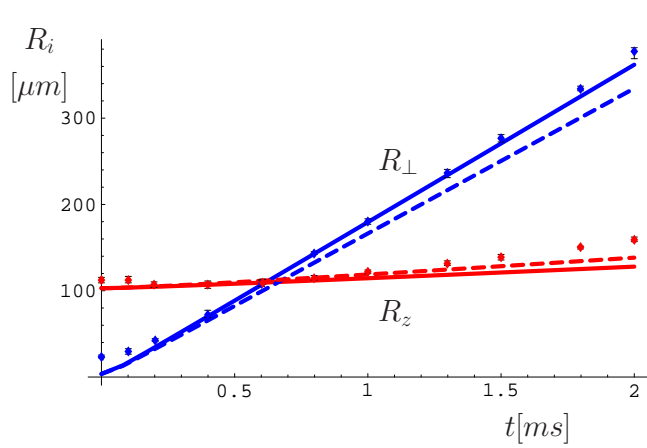
$$\begin{aligned}\dot{E} = & -\frac{1}{2} \int d^3x \eta(x) \left( \partial_i v_j + \partial_j v_i - \frac{2}{3} \delta_{ij} \partial_k v_k \right)^2 \\ & - \int d^3x \zeta(x) (\partial_i v_i)^2 - \frac{1}{T} \int d^3x \kappa(x) (\partial_i T)^2\end{aligned}$$

Have  $\zeta = 0$  and  $T(x) = \text{const.}$  Universality implies

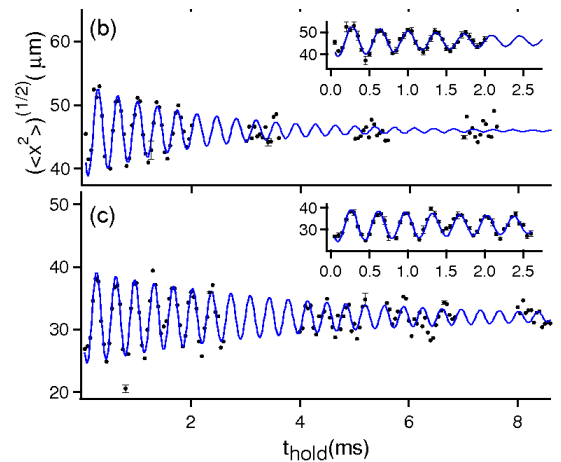
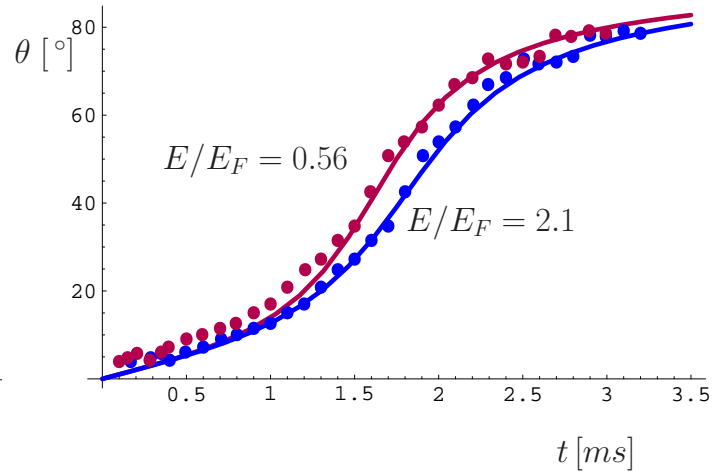
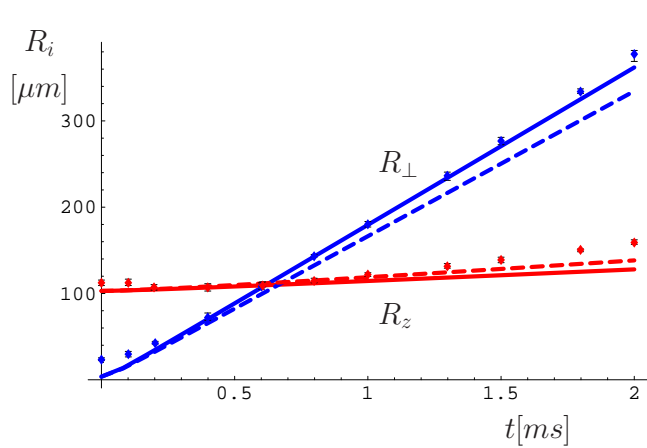
$$\eta(x) = s(x) \alpha_s \left( \frac{T}{\mu(x)} \right)$$

$$\int d^3x \eta(x) = S \langle \alpha_s \rangle$$

# Dissipation



# Dissipation



$$\left. \begin{array}{l} (\delta t_0)/t_0 \\ (\delta a)/a \end{array} \right\} = \left\{ \begin{array}{l} 0.008 \\ 0.024 \end{array} \right\} \left( \frac{\langle \alpha_s \rangle}{1/(4\pi)} \right) \left( \frac{2 \cdot 10^5}{N} \right)^{1/3} \left( \frac{S/N}{2.3} \right) \left( \frac{0.85}{E_0/E_F} \right)$$

$t_0$ : “Crossing time” ( $b_{\perp} = b_z$ ,  $\theta = 45^\circ$ )

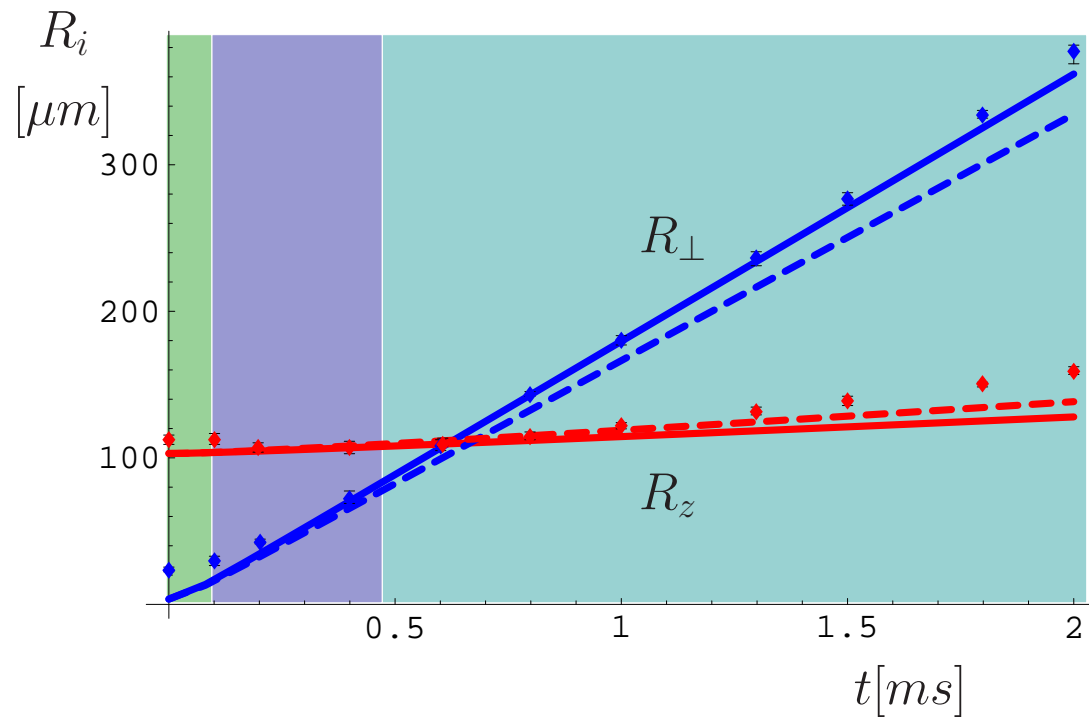
$a$ : amplitude

# Time Scales

dissipative

hydro/free streaming

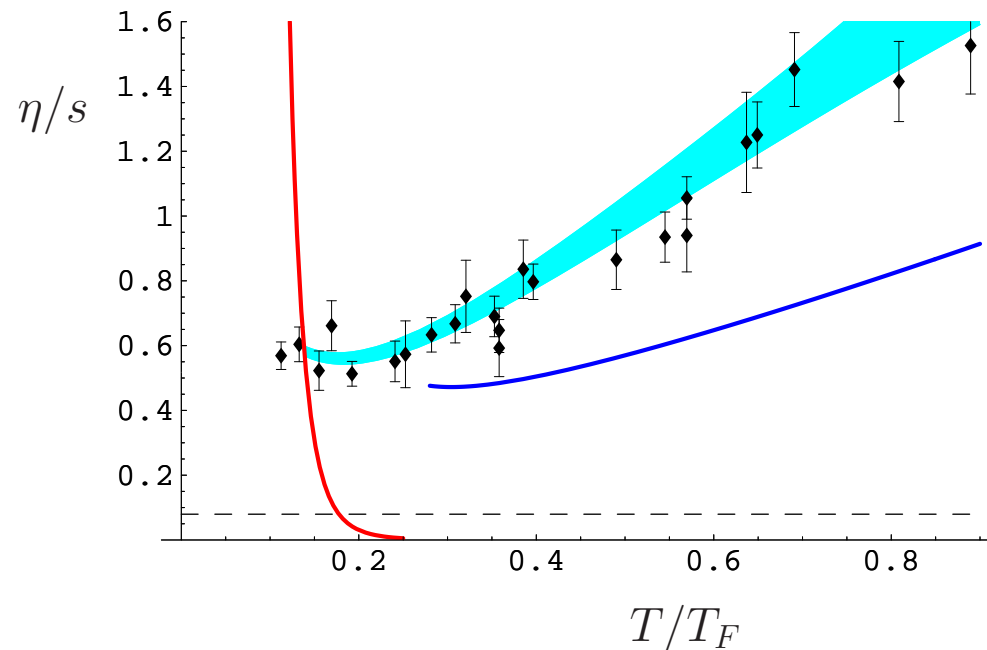
ballistic



# Collective modes: Small viscous correction exponentiates

$$a(t) = a_0 \cos(\omega t) \exp(-\Gamma t)$$

$$\frac{\eta}{s} = \frac{3}{4} (3N\lambda)^{1/3} \left( \frac{\Gamma}{\omega_{\perp}} \right) \left( \frac{E_0}{E_F} \right) \left( \frac{N}{S} \right)$$



# Limitations of Scaling Flows

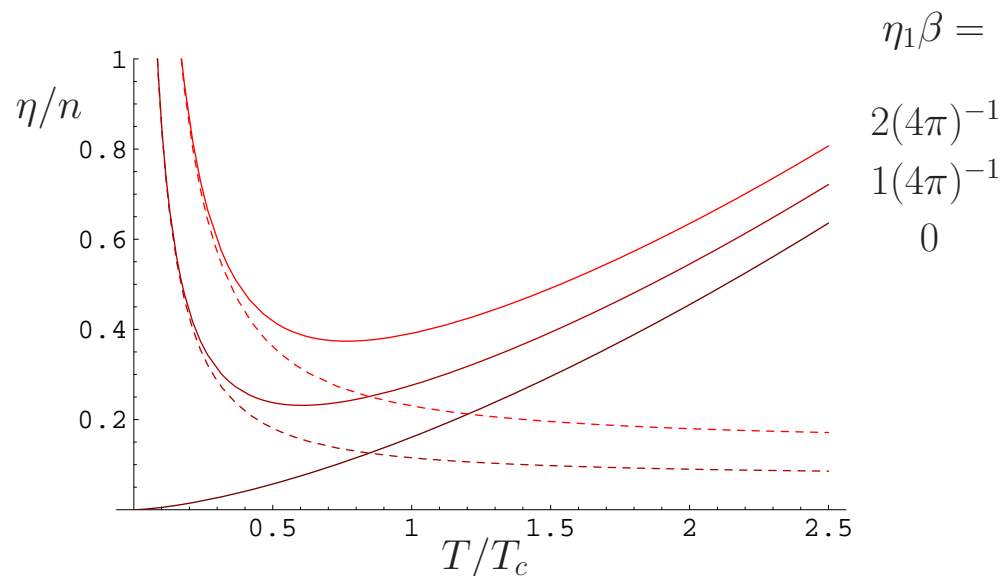
Simple model for  $\eta(n, T)$  [where  $P = (n^{5/3}/m)f(mT/n^{2/3})$ ]

$$\eta(n, T) = \eta_0(mT)^{3/2} + \eta_1 \frac{n^{5/3}}{mT} f\left(\frac{mT}{n^{2/3}}\right),$$

Find exact scaling solutions of the Navier Stokes equation

But:  $\eta_0$  completely unconstrained by data

$$\nabla_j [\eta_0(mT)^{3/2} (\nabla_i v_j + \dots)] = 0$$



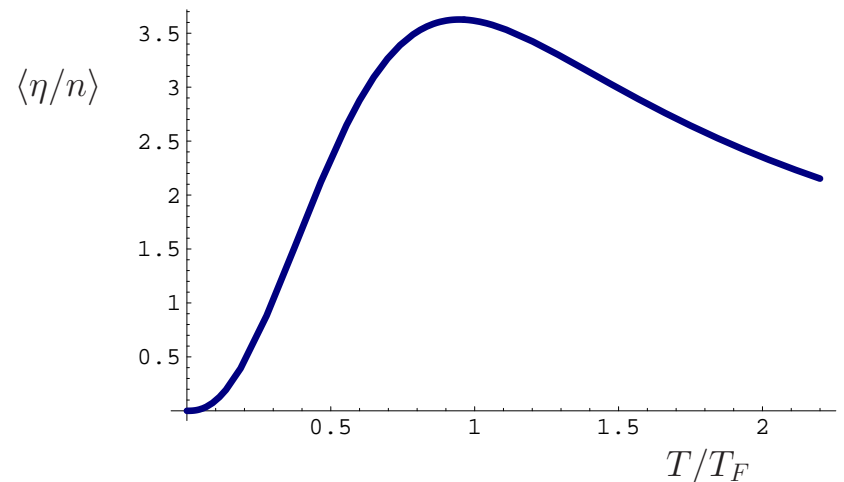
## Relaxation Time Model

In real systems stress tensor does not relax to Navier-Stokes form instantaneously. Consider

$$\tau_R \left( \frac{\partial}{\partial t} - \vec{v} \cdot \vec{\nabla} \right) \delta\Pi_{ij} = \delta\Pi_{ij} - \eta \left( \partial_i v_j + \partial_j v_i - \frac{2}{3} \delta_{ij} \partial \cdot v \right)$$

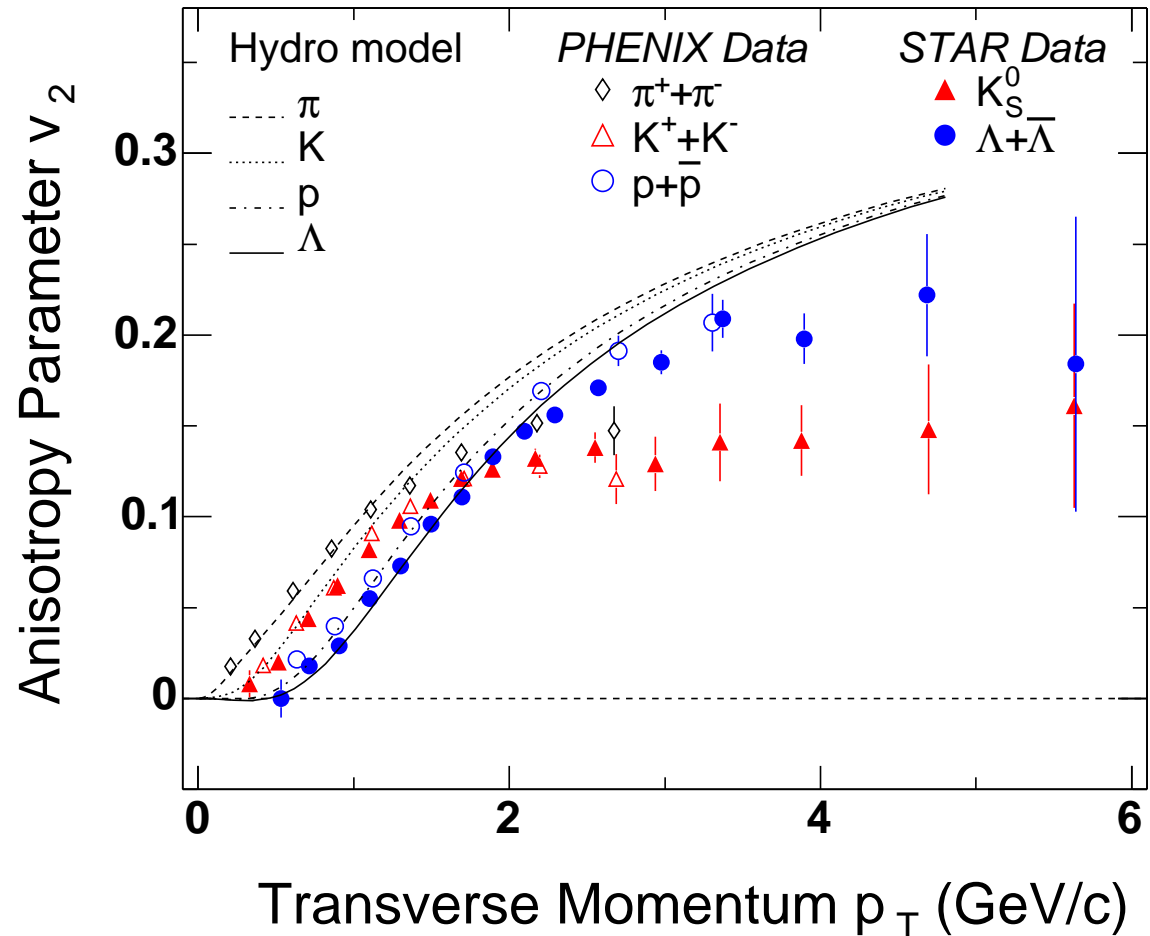
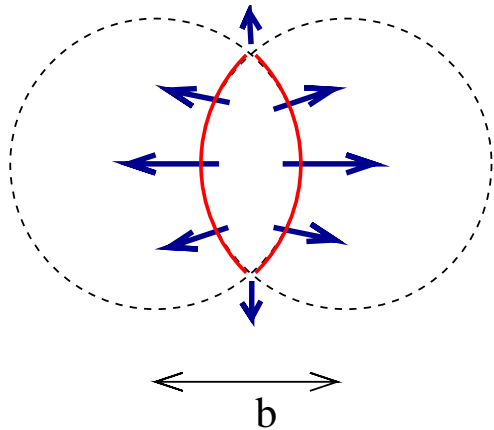
In kinetic theory  $\tau_R \simeq (\eta/n) T^{-1}$

- dissipation from  $\eta \sim (mT)^{3/2}$
- modified  $T$  dependence
- modified  $N$  scaling
- expansion  $\leftrightarrow$  coll oscillations



### III. Elliptic Flow (QGP)

Hydrodynamic expansion converts  
coordinate space  
anisotropy  
to momentum space  
anisotropy



source: U. Heinz (2005)

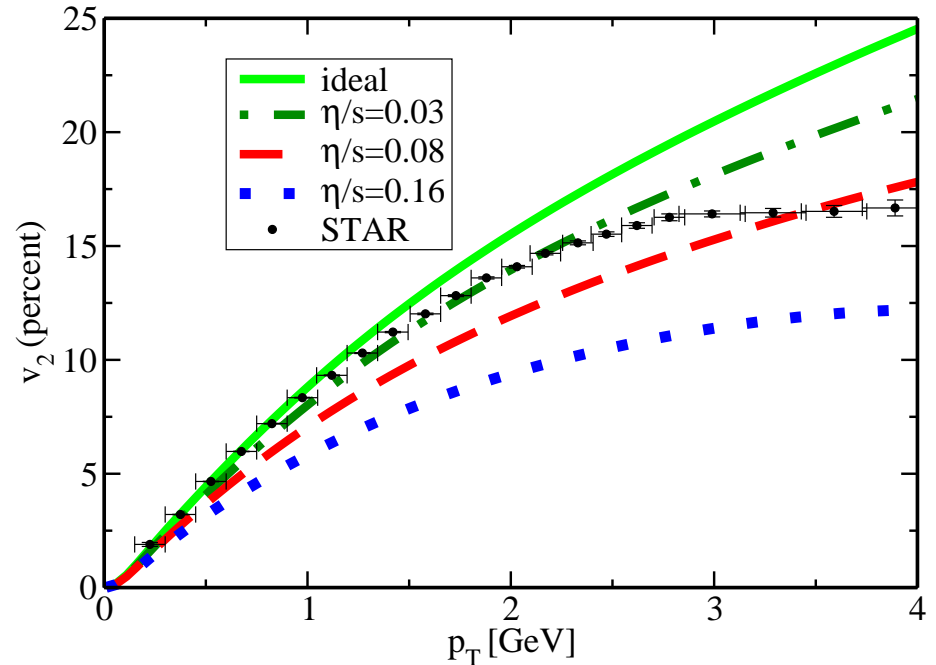
# Viscosity and Elliptic Flow

Consistency condition  $T_{\mu\nu} \gg \delta T_{\mu\nu}$   
(applicability of Navier-Stokes)

$$\frac{\eta + \frac{4}{3}\zeta}{s} \ll \frac{3}{4}(\tau T)$$

Danielewicz, Gyulassy (1985)

Very restrictive for  $\tau < 1$  fm



Romatschke (2007), Teaney (2003)

Many questions: Dependence on initial conditions, freeze out, etc.

## Outlook

Too early to declare a winner.

$$\eta/s \simeq 0.8 \text{ (He)}, \quad \eta/s \leq 0.5 \text{ (CA)}, \quad \eta/s \leq 0.5 \text{ (QGP)}$$

Other experimental constraints, more analysis needed.

Kinetic theory: o.k. in He (all  $T$ ), o.k. close to  $T_c$  in CA, QGP?

New theory tools: AdS/Cold Atom correspondence? Field theory approaches in cross over regime (large  $N$ , epsilon expansions, ...)