Renormalization group for symmetry-broken phases near quantum critical points

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1. Introduction
2. ϕ^4-theory
3. Full potential RG

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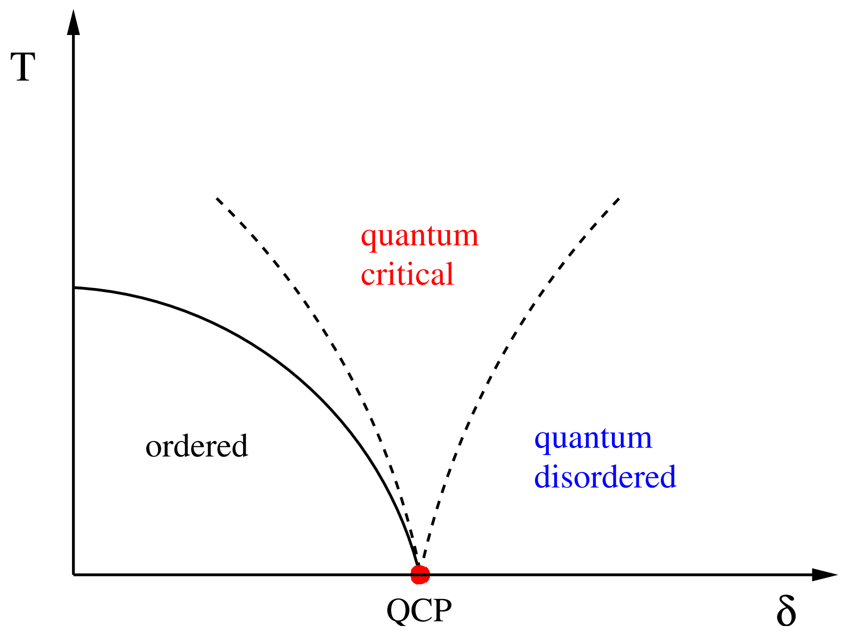
1. Introduction

Quantum phase transitions:

Phase transition at $T = 0$ driven by control parameter δ (pressure, density, ...)



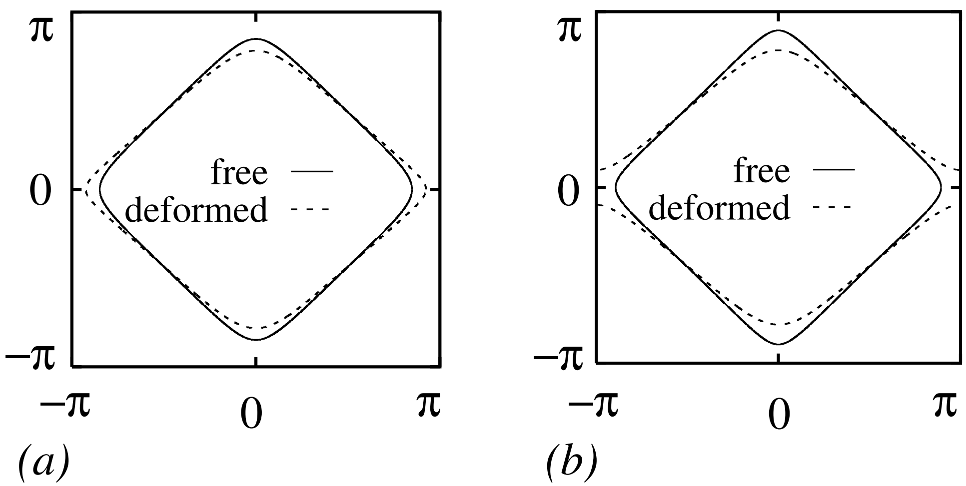
If order parameter vanishes *continuously* at transition: quantum critical point.

Quantum fluctuations lead to unusual physical properties.

Quantum phase transitions in metals:

non-Fermi liquid behavior in quantum critical regime

Example: d-wave Pomeranchuk (nematic) transition



Spontaneous breaking of tetragonal symmetry

Halboth, WM '00

Yamase, Kohno '00

Order parameter $n_d = \sum_{\mathbf{k}} d_{\mathbf{k}} \langle n_{\mathbf{k}} \rangle$ where $d_{\mathbf{k}} = \cos k_x - \cos k_y$

Realization of "nematic" electron liquid ($\rightarrow$ Kivelson et al. '98)

Experimental evidence for nematic phase in
$Sr_3Ru_2O_7$ (Mackenzie group) and YBCO (Keimer group)

Non-Fermi liquid behavior near d-wave Pomeranchuk instability:

Scattering at Fermi surface fluctuations leads to large decay rates

- At quantum critical point ($T = 0$, $\xi = \infty$):

$$\mathrm{Im}\Sigma(\mathbf{k}_F, \omega) \;\propto\; d^2_{\mathbf{k}_F} |\omega|^{2/3} \quad \text{for} \;\; \omega \to 0$$

WM, Rohe, Andergassen '03

 * large anisotropic decay rate of single-particle excitations

 * maximal near van Hove points,
 minimal near diagonal in Brillouin zone: "cold spots"

 $\Rightarrow$ no quasi-particles away from Brillouin zone diagonal

- Decay rate for DC transport in quantum critical regime:

$$\gamma^{\mathrm{tr}}_{\mathbf{k}_F} \propto d^2_{\mathbf{k}_F} T$$

Dell'Anna, WM '07 — as observed by Hussey '06 in cuprates

Question 1: $T_c(\delta)$ near QCP ?

Mean field theory: $T_c^{\mathrm{MF}}(\delta) \propto |\delta - \delta_{\mathrm{qc}}|^{1/2}$ independent of dimension and symmetry

Millis '93:

$T_c(\delta)$ estimated by Ginzburg temperature $T_G(\delta)$ in symmetric phase (for d above lower critical dimension)

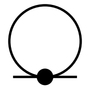
δ-dependence of $T_c(\delta)$ differs strongly from $T_c^{\mathrm{MF}}(\delta)$

Here:

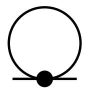
compute $T_c(\delta)$ directly from condition $\xi(\delta, T_c) \to \infty$ (ξ correlation length)

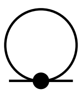
must enter non-Gaussian fluctuation regime near classical phase transition

Sachdev '97: $T_c(\delta)$ et al. from general scaling arguments

$T_c(\delta)$ from tadpole  ?

Consider $T > 0$ (classical phase transition)

$$\xi^{-2}(T) = a\,(T - T_c^{\mathrm{MF}}) +$$ 

 $$\propto \; u\,T \int \frac{d^d p}{\xi^{-2} + p^2} \; \propto \; T \log \xi(T)$$ for $d = 2$

$\xi(T) = \infty$ only at $T = 0$ in two dimensions !

This is *not* Mermin-Wagner (happens also for discrete symmetry) !

Question 2:

What if phase transition is first order in mean-field theory?

Can order parameter fluctuations make it continuous?

2. ϕ^4-theory for symmetry-broken phase

Case of discrete symmetry (no Goldstone mode)

Real scalar order parameter ϕ, integrate out fermions $\Rightarrow$

Hertz action:

$$\mathcal{S}[\phi] = \frac{T}{2} \sum_{\omega_n} \int \frac{d^d q}{(2\pi)^d} \, \phi_q \left(\frac{|\omega_n|}{|\mathbf{q}|^{z-2}} + \mathbf{q}^2 \right) \phi_{-q} + \mathcal{U}[\phi]$$

dynamical exp. $z \geq 2$

Pomeranchuk: $z = 3$

Potential $\mathcal{U}[\phi]$ in symmetry broken regime:

$$\mathcal{U}[\phi] = \frac{u}{4!} \int_0^{1/T} d\tau \int d^d r \left[\phi^2(\mathbf{r},\tau) - {\phi_0}^2 \right]^2$$

double well

$$= \int_0^{1/T} d\tau \int d^d r \left[u \, \frac{\phi'^4}{4!} + \sqrt{3\, u\, \delta} \, \frac{\phi'^3}{3!} + \delta \, \frac{\phi'^2}{2!} \right]$$

$\phi' = \phi - \phi_0$

$\delta = u {\phi_0}^2 / 3$

Remarks:

- Assume that fermions can be integrated out safely (no singular interactions)
- Not suitable for symmetry broken phases with gaps

Functional renormalization group

Exact functional flow equation for effective action $\Gamma^\Lambda[\phi]$: Wetterich '93

$$\partial_\Lambda \Gamma^\Lambda[\phi] = \frac{1}{2} \mathrm{tr} \frac{\partial_\Lambda R^\Lambda}{\Gamma^{(2)}[\phi] + R^\Lambda} \quad \text{where} \quad \Gamma^{(2)}[\phi] = \frac{\delta^2 \Gamma^\Lambda}{\delta\phi\, \delta\phi}$$

Regulator function: $R^\Lambda(\mathbf{q}) = Z^\Lambda(\Lambda^2 - \mathbf{q}^2)\Theta(\Lambda^2 - \mathbf{q}^2)$ acts as momentum cutoff

Z^Λ = wave function renormalization

initial condition: $\Gamma^{\Lambda_0}[\phi] = S[\phi]$, Λ_0 ultraviolet cutoff

final (quantum) effective action: $\Gamma[\phi] = \lim_{\Lambda \to 0} \Gamma^\Lambda[\phi]$

Approximate ansatz for effective action:

$$\Gamma^\Lambda[\phi] = \frac{T}{2}\sum_{\omega_n}\int \frac{d^dq}{(2\pi)^d}\,\phi_q \left(Z_\omega^\Lambda \frac{|\omega_n|}{|\mathbf{q}|^{z-2}} + Z^\Lambda \mathbf{q}^2 \right) \phi_{-q} + \mathcal{U}^\Lambda[\phi]$$

where

$$\mathcal{U}^\Lambda[\phi] = \frac{u^\Lambda}{4!}\int_0^{1/T} d\tau \int d^dr \left[\phi^2(\mathbf{r},\tau) - \phi_0^{\Lambda 2}\right]^2$$

same form as bare action, but flowing parameters

Mass (inverse susceptibility):

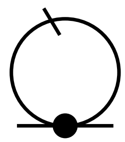
$$\delta^\Lambda = u^\Lambda(\phi_0^\Lambda)^2/3$$

Correlation length:

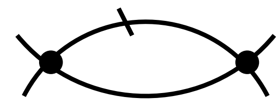
$$\xi^\Lambda = \sqrt{Z^\Lambda/\delta^\Lambda} \quad \xrightarrow{\Lambda\to 0} \quad \xi$$

Flow equations for $\rho_0 = \phi_0^2$, u , and Z :

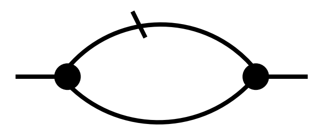
$$\partial_\Lambda \rho_0 = 3T \sum_{\omega_n} \int \frac{d^d q}{(2\pi)^d} \dot{R}(\mathbf{q})\, G^2(\mathbf{q}, \omega_n)$$



$$\partial_\Lambda u = 3\, u^2\, T \sum_{\omega_n} \int \frac{d^d q}{(2\pi)^d} \dot{R}(\mathbf{q})\, G^3(\mathbf{q}, \omega_n)$$



$$\partial_\Lambda Z = \frac{3u\delta}{2d} \Delta_{\mathbf{p}}\, T \sum_{\omega_n} \int \frac{d^d q}{(2\pi)^d} \dot{R}(\mathbf{q})\, G^2(q)\, G(p+q) \Big|_{\mathbf{p}=0}$$



where $\quad G^{-1}(\mathbf{q}, \omega_n) = Z_\omega \frac{\omega_n}{|\mathbf{q}|^{z-2}} + Z\mathbf{q}^2 + \delta + R(\mathbf{q})$

dashed lines represent $G^2 \dot{R} = -\partial_\Lambda G$ $\quad$ (∂_Λ acts only on R)

Only one-loop integrals; flow of Z_ω negligible

Rescaled variables (to eliminate Λ and Z in flow equations):

$$\tilde{\mathbf{q}} = \mathbf{q}/\Lambda \qquad \tilde{T} = \frac{1}{Z\Lambda^z} T$$

$$\tilde{\delta} = \frac{\delta}{Z\Lambda^2} \qquad \tilde{u} = \frac{T}{Z^2\Lambda^{4-d}} u$$

anomalous dimension $\eta = -\frac{d \log Z}{d \log \Lambda}$

dimensionless flow parameter:

$s = -\log(\Lambda/\Lambda_0)$, Λ_0 ultraviolet cutoff

Solution at zero temperature:

For $d + z \geq 4$ stable Gaussian fixed point at $\tilde{\delta} = 0$,
corresponding to a bare $\delta_{\rm qc} > 0$.

Order parameter and correlation length at $T = 0$ near QCP:

$$\phi_0 \propto \sqrt{\delta - \delta_{\rm qc}} \qquad \xi \propto \frac{1}{\sqrt{\delta - \delta_{\rm qc}}}$$

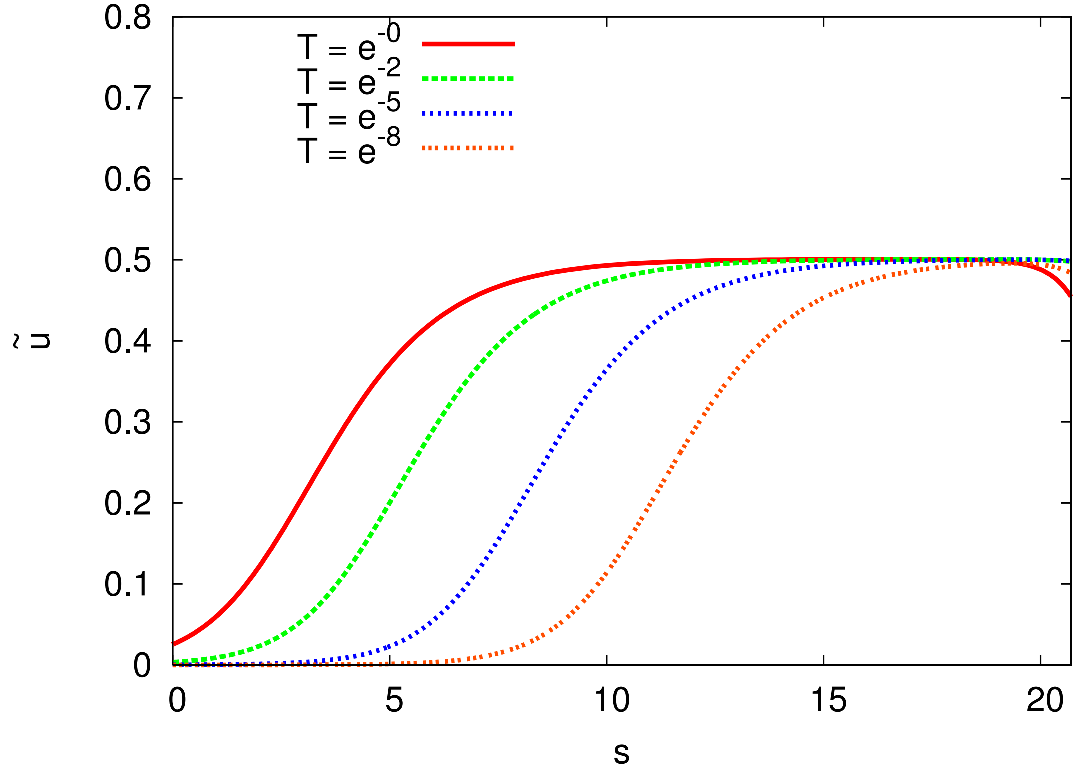
Mean-field behavior (as expected)

Flow at finite temperature:

Focus on temperatures at and slightly below $T_c(\delta)$

Compute $T_c(\delta)$ and $T_G(\delta)$

Flow of $\tilde{u}$ for $T \approx T_c(\delta)$ as a function of $s = -\log(\Lambda/\Lambda_0)$

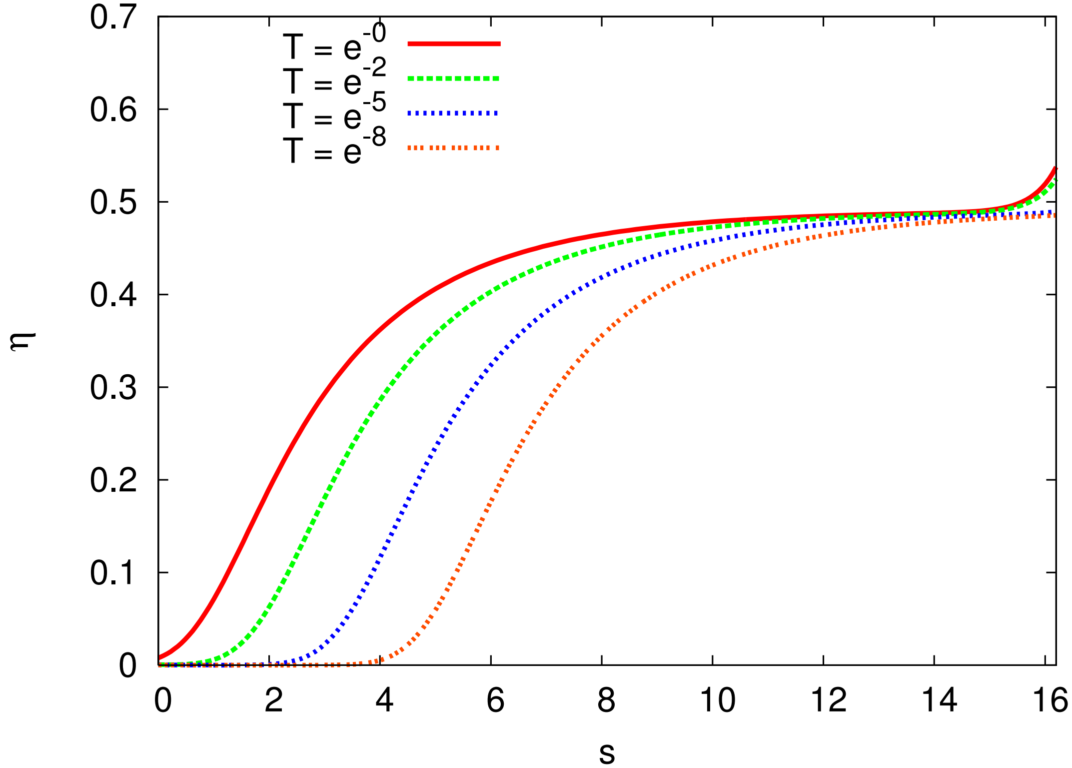


$z = 3$

$d = 3$

Plateaus correspond to non-Gaussian classical fixed points

Flow of η for $T \approx T_c(\delta)$

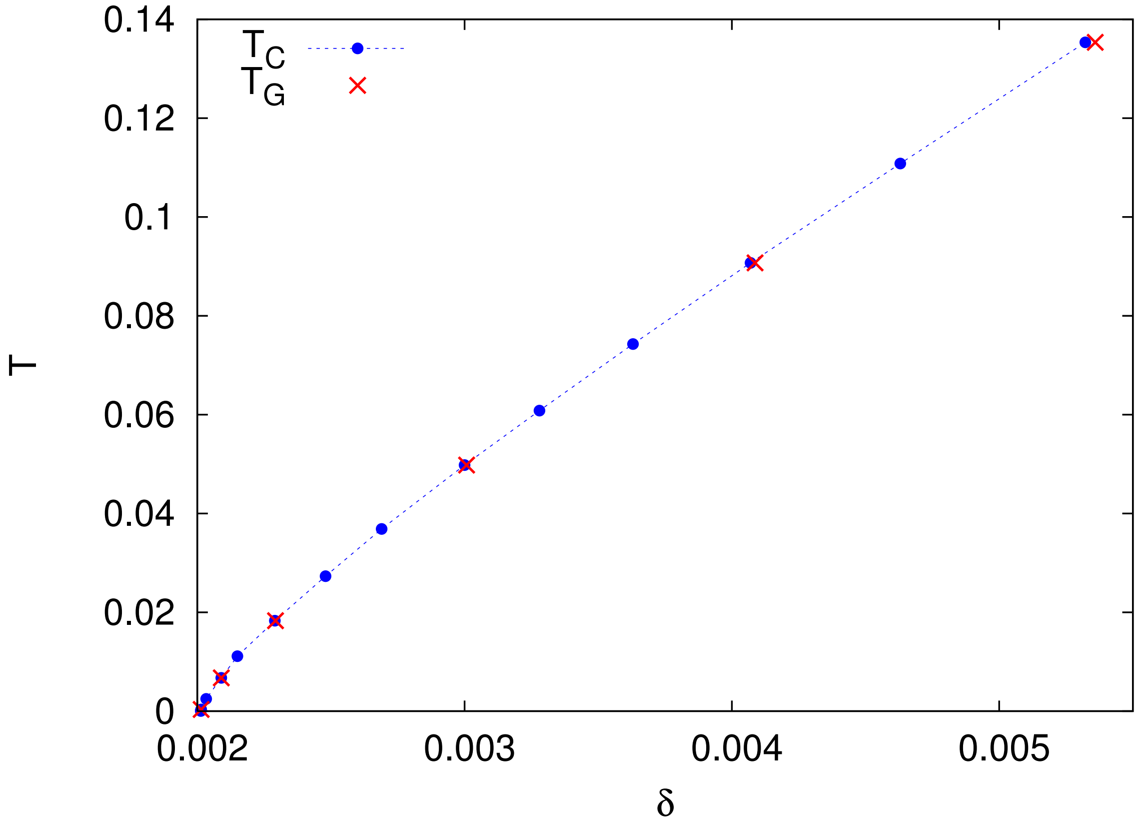


$z = 3$

$d = 2$

Sizable anomalous dimension at classical fixed point in two dimensions (overestimated by factor 2 compared to exact Onsager value)

$T_c(\delta)$ and $T_G(\delta)$ for $z = 3$ in $d = 3$:



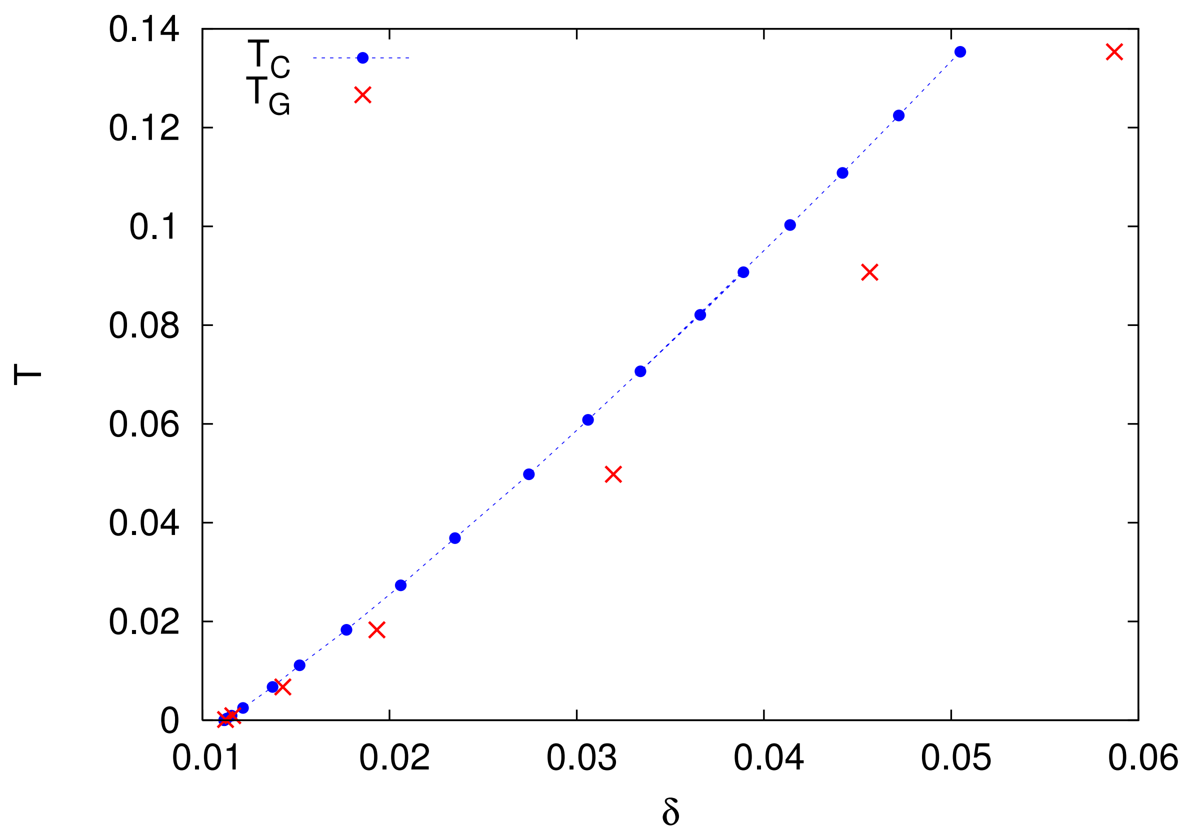
T_c and T_G
almost coincide

$T_c \propto (\delta - \delta_{\rm qc})^{3/4}$

$T_c(\delta)$ in agreement with general expression $T_G(\delta) \propto (\delta - \delta_{\rm qc})^{\psi}$

with shift exponent $\psi = \frac{z}{d+z-2}$, valid for $d > 2$ (Millis '93)

$T_c(\delta)$ and $T_G(\delta)$ for $z = 3$ in $d = 2$:

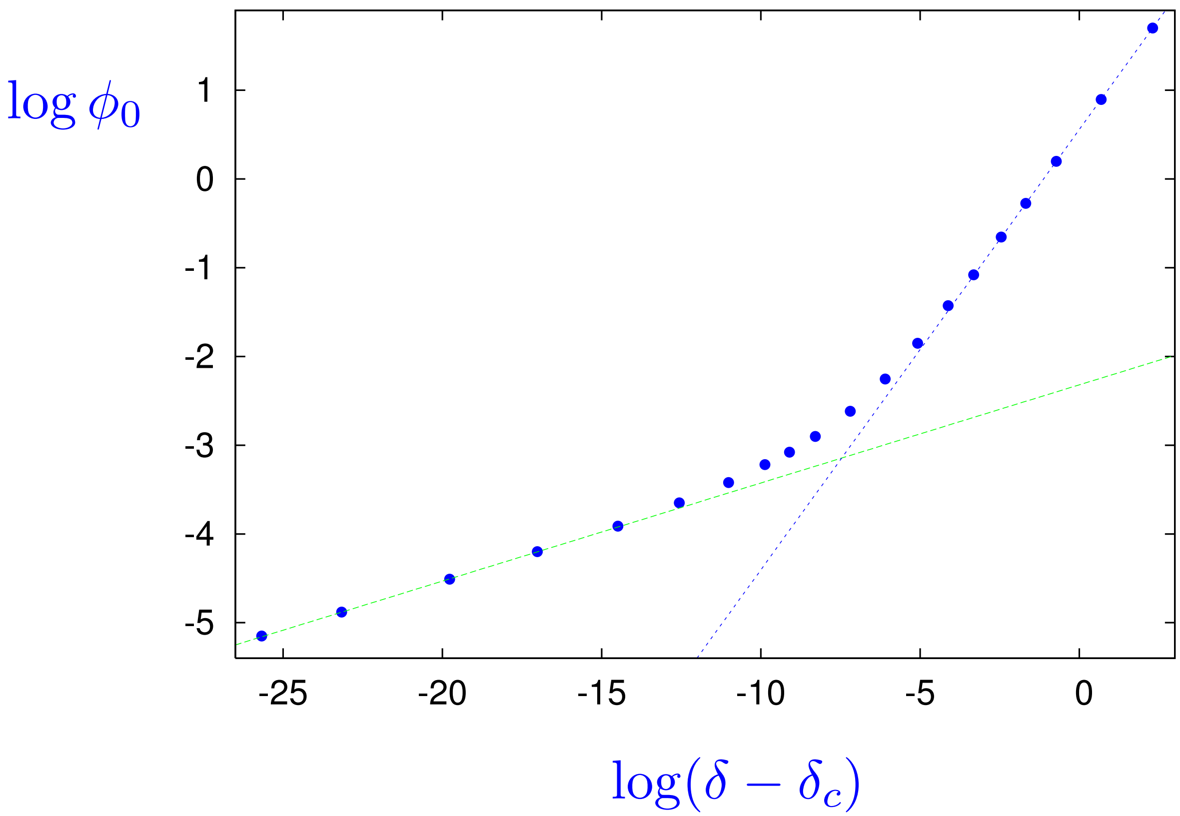


Sizable window between T_c and T_G

$T_c \log T_c \propto \delta - \delta_{\rm qc}$

$T_c(\delta)$ in agreement with $T_G \log T_G \propto \delta - \delta_{\rm qc}$ obtained from Ginzburg criterion (Millis '93)

Computation of Ginzburg points:



Order parameter versus control parameter at fixed temperature

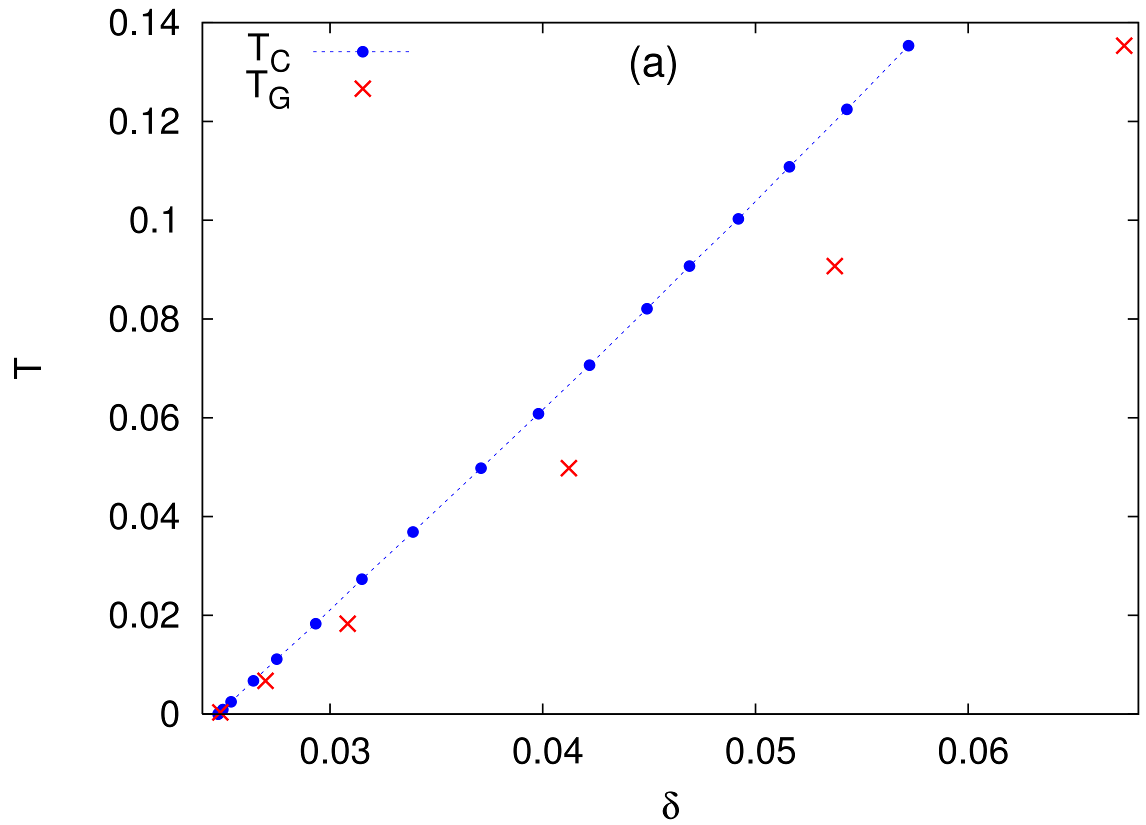
$z = 3$

$d = 2$

Mean-field behavior at large $\delta - \delta_c$, non-Gaussian scaling at small $\delta - \delta_c$

Intersection of straight lines determines Ginzburg value δ_G

Results for $z = 2$ similar and also in agreement with Millis



T_c and T_G
for $z = 2$, $d = 2$

Sizable window between T_c and T_G in two dimensions

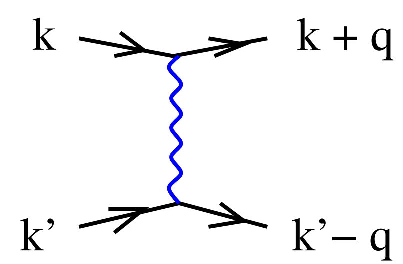
3. Full potential RG

Problem:

- Nematic transition on square lattice typically first order at low T
- Expansion of Landau energy in powers of ϕ not possible

Phenomenological 2D lattice model: WM, Rohe, Andergassen '03

$$H = H_{\rm kin} + \frac{1}{2L} \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} f_{\mathbf{k}\mathbf{k}'}(\mathbf{q})\, n_{\mathbf{k}}(\mathbf{q})\, n_{\mathbf{k}'}(-\mathbf{q})$$



where $n_{\mathbf{k}}(\mathbf{q}) = \sum_\sigma c^\dagger_{\mathbf{k}+\mathbf{q},\sigma}\, c_{\mathbf{k},\sigma}$

and only small momentum transfers $\mathbf{q}$ contribute (forward scattering)

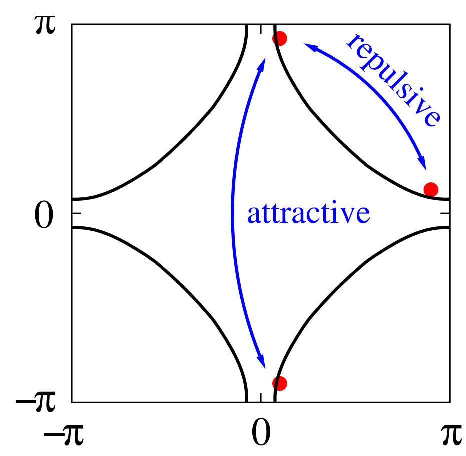
$H_{\rm kin}$ tight-binding kinetic energy from hopping t, t'

Interaction with d-wave attraction:

$f_{\mathbf{k}\mathbf{k}'}(\mathbf{q}) = -g(\mathbf{q})\, d_{\mathbf{k}}\, d_{\mathbf{k}'}$

with $d_{\mathbf{k}} = \cos k_x - \cos k_y$ and $g(\mathbf{q}) > 0$

yields Pomeranchuk instability



Mean-field theory:

Grand canonical potential

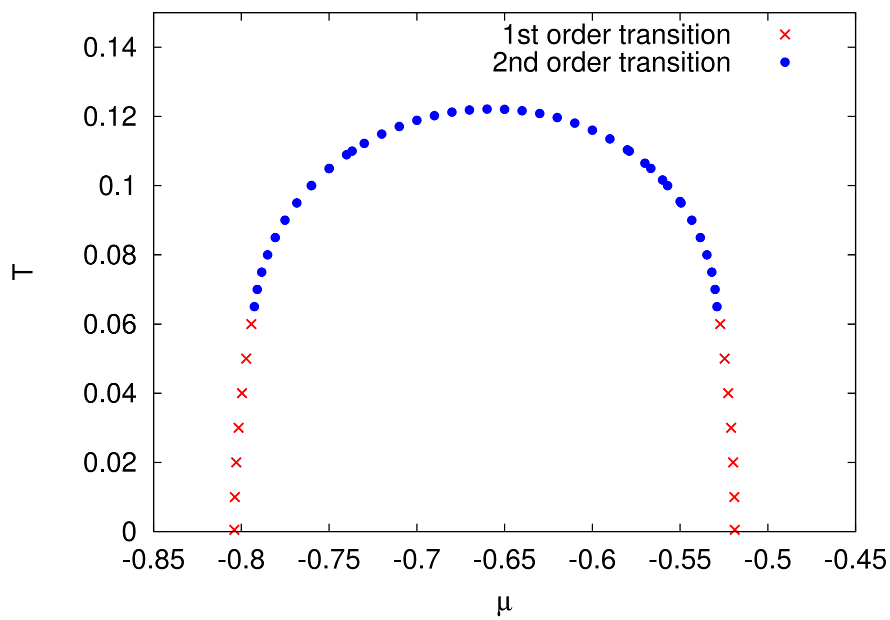
$$\frac{\Omega}{L} = \frac{\phi^2}{2g} - \frac{2T}{L}\sum_{\mathbf{k}} \ln\left(1 + e^{-(\epsilon_{\mathbf{k}} - \phi d_{\mathbf{k}} - \mu)/T}\right) \quad \text{where} \quad \phi = g\sum_{\mathbf{k}} d_{\mathbf{k}} \langle n_{\mathbf{k}} \rangle$$

Transition first order

at low temperatures

Khavkine et al. '04

Yamase et al. '05



Coefficients of expansion $\Omega(\phi) = \sum_{n \geq 0} \frac{a_{2n}}{(2n)!} \phi^{2n}$

all negative for $n \geq 2$ at low temperatures !

Jakubczyk '09: ϕ^6-theory with $a_6 > 0$

- Fluctuations enhance a_4
- First order mean-field transition may become continuous

Functional RG with full potential:

Ansatz for effective action:

$$\Gamma^\Lambda[\phi] = \frac{T}{2} \sum_{\omega_n} \int \frac{d^d q}{(2\pi)^d} \, \phi_q \left(Z_\omega^\Lambda \frac{|\omega_n|}{|\mathbf{q}|^{z-2}} + Z^\Lambda \mathbf{q}^2 \right) \phi_{-q} + \mathcal{U}^\Lambda[\phi]$$

where now

$$\mathcal{U}^\Lambda[\phi] = \int_0^{1/T} d\tau \int d^d r \, U^\Lambda(\phi(\mathbf{r},\tau))$$ with arbitrary function $U^\Lambda(\phi)$

Nematic transition:

dynamical exponent $z = 3$

Initial condition for potential: $U^{\Lambda_0}(\phi) = L^{-1}\Omega(\phi)$ (mean-field pot.)

Flow equations:

$$\partial_\Lambda U^\Lambda(\phi) = \frac{1}{2}\int_q \dot{R}^\Lambda(q) G^\Lambda(q;\phi)$$

$$G^\Lambda(q;\phi) = \left[Z_\omega^\Lambda \frac{|\omega_n|}{|\mathbf{q}|^{z-2}} + Z^\Lambda \mathbf{q}^2 + \partial_\phi^2 U^\Lambda(\phi) + R^\Lambda(q) \right]^{-1}$$

Partial differential equation!

Flow of Z-factors from flow of two-point vertex

$$\partial_\Lambda \Gamma_2^\Lambda(q;\phi) = \left[\partial_\phi^3 U(\phi)\right]^2 \int_p \dot{R}^\Lambda(p) \left[G^\Lambda(p;\phi)\right]^2 G^\Lambda(p+q;\phi)$$
$$- \frac{1}{2}\,\partial_\phi^4 U(\phi) \int_p \dot{R}^\Lambda(p) \left[G^\Lambda(p;\phi)\right]^2$$

Flow of Z_ω^Λ negligible, flow of Z^Λ dominated by classical fluctuations

Flow of potential $U^\Lambda(\phi)$:

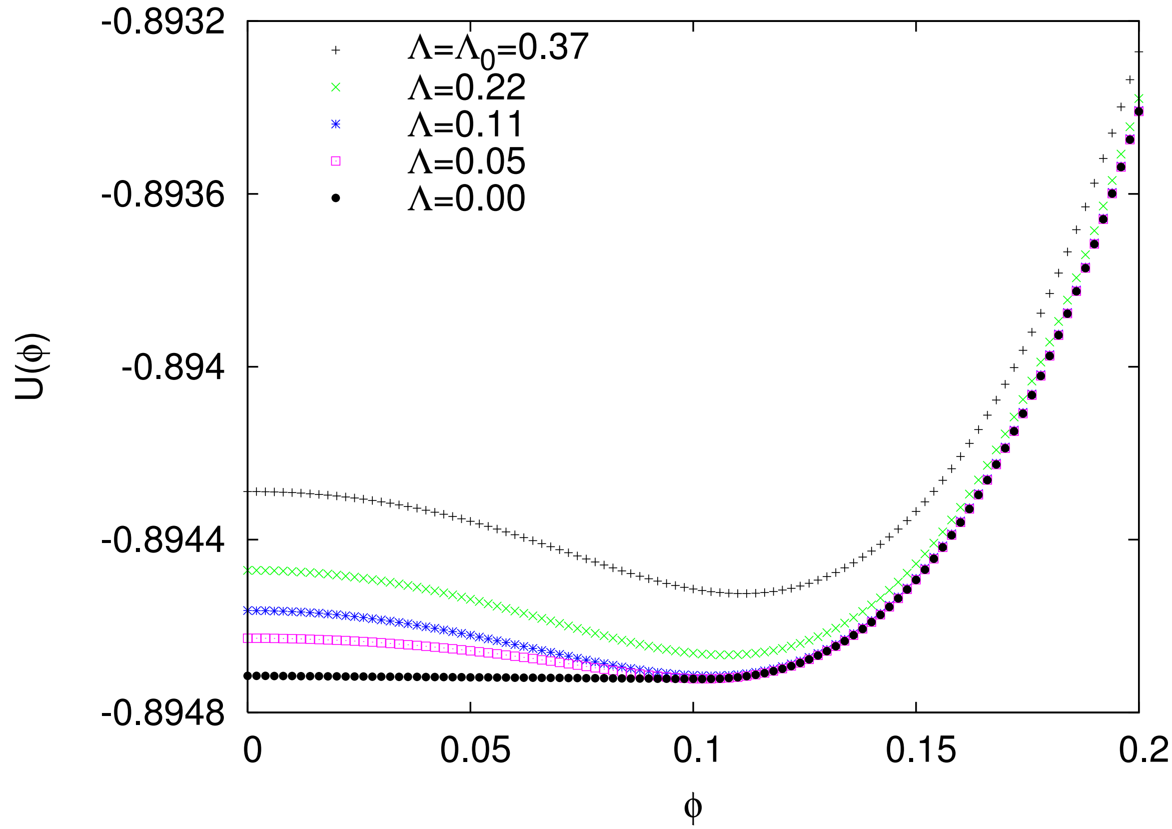
Parameters:

$t = 1$, $t' = -1/6$

$g = 0.8$

$Z^{\Lambda_0} = 10$

$Z_\omega^{\Lambda_0} = 1$



For $\Lambda = 0$ flat for $\phi \in [0, \phi_0]$ as required by convexity.

Phase diagram:

Critical temperature $T_c(\mu)$

for

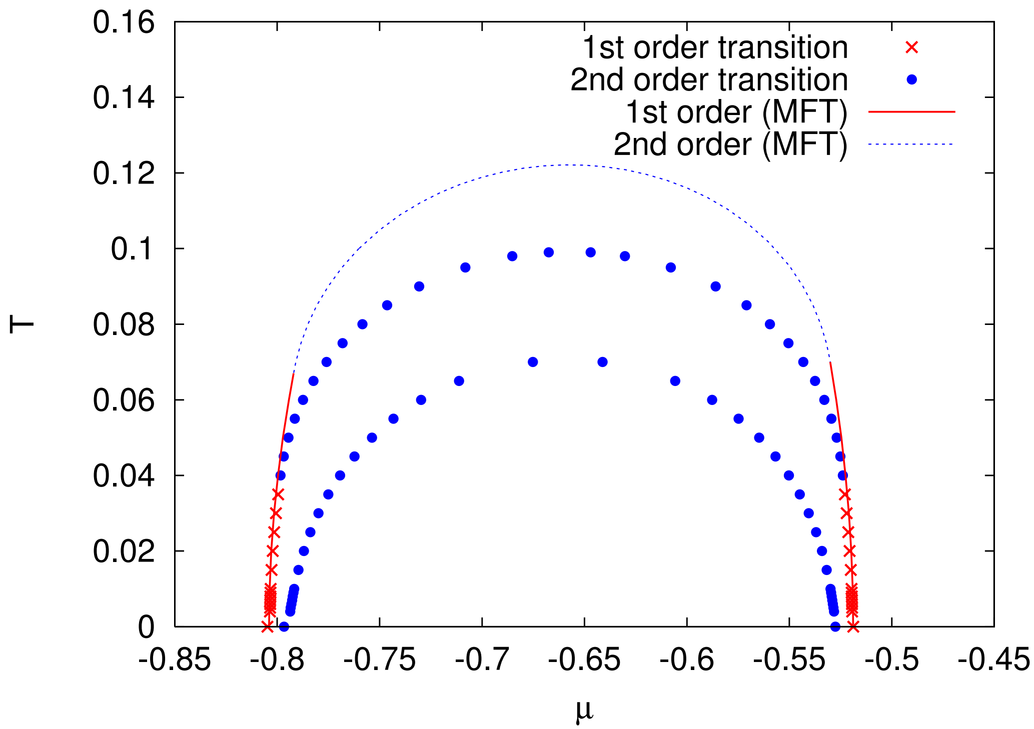
$\Lambda_0 = 0.37$

and

$\Lambda_0 = 1.00$

compared to

mean-field result

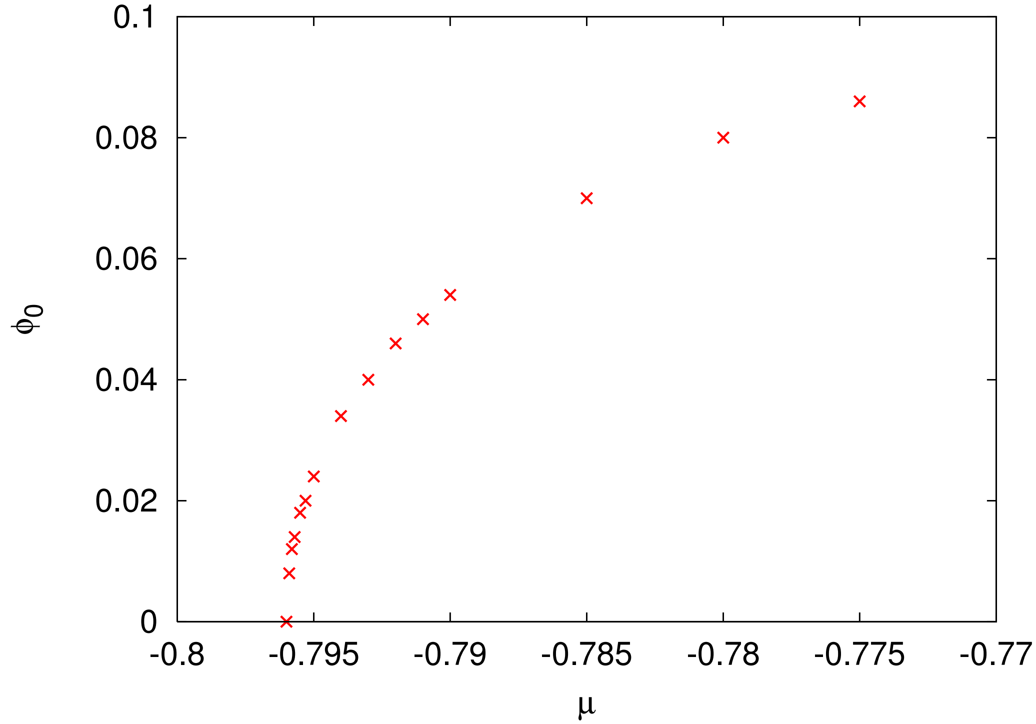


Fluctuations suppress T_c (as expected).

Continuous transition down to $T = 0$ can be realized!

Order parameter ϕ_0:

$\phi_0(\mu)$ at $T = 0$
for $\Lambda_0 = 1.00$



Consistent with Gaussian quantum critical point

Summary:

- Simple extension of Hertz-Millis theory to phases with broken discrete symmetry, accessing also the non-Gaussian fluctuation regime.

- Ginzburg criterium yields very accurate estimate of $T_c(\delta)$ in 3D, and qualitatively correct behavior of $T_c(\delta)$ for discrete symmetry breaking in 2D.

- Sizable window between $T_G(\delta)$ and $T_c(\delta)$ in 2D even for discrete symmetry breaking.

- Functional RG with full potential $U(\phi)$ can deal with cases where expansion in powers of ϕ is not possible.

- Fluctuations can turn a first order mean-field transition to a nematic phase continuous.