Holographic superconductors

Sean Hartnoll

Harvard University

Work in collaboration with

Chris Herzog and Gary Horowitz :	0801.1693, 0810.1563.
Frederik Denef :	0901.1160.
Frederik Denef and Subir Sachdev :	0908.????.

RPMBT15 – July 09 – OSU

Plan of talk

Background

1. What is applied string theory?
2. Basics of the holographic correspondence.

Holographic superconductors

1. Ingredients for a holographic superconductor
2. Black hole instabilities
3. Electrical conductivity
4. Landscape of superconducting membranes

Background

1. What is applied string theory?
2. Basics of the holographic correspondence.

Applied string theory

- A large class of exotic strongly interacting QFTs are known to be described by gravitational duals.

- Introduces a new, geometrical, vocabulary for thinking about QFTs.

- Philosophy: Rather than focus on specific theories, ask what characteristic phenomena emerge within this framework?

- What do these theories look like at finite charge density?

Why am I giving this talk?

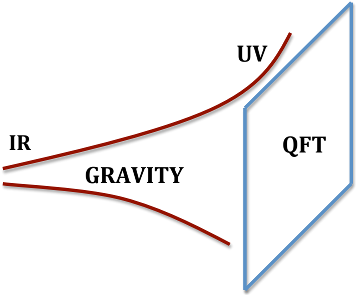
- Allows computations directly at strong coupling.
  - Correlators of operators in low energy effective field theory.

- Dispense of 'conceptual baggage' from weak coupling.
  - cf. ε or vector large N expansions: compute at weak coupling.
  - in general no quasiparticles, no 'single-particle' Green's functions, often no weakly coupled 'UV' Lagrangian.

- Positive interaction with heavy ion physics.
  - Computation of $\eta/s \rightarrow$ elliptic follow consistent with gauge theory.
  - Simple computations of real-time quantum dynamics at finite temperature and density.

The holographic correspondence geometrises Wilson

- Renormalisation group equations are local in the energy scale.
- Holographic correspondence: add this scale as an extra curved spacetime dimension. Einstein's equations are the RG equations.



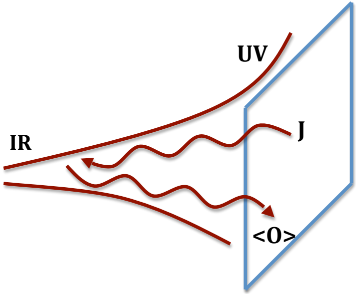
- e.g. conformal invariance $\longrightarrow$ Anti-de Sitter space

$$ds^2 = r^2 \left(-dt^2 + dx^2 + dy^2\right) + \frac{dr^2}{r^2} \,.$$

Sources are boundary conditions

- Every field ϕ in the bulk corresponds to an operator $\mathcal{O}$ in the QFT.
- The boundary value of the field sources the dual operator

$$\langle e^{\int \mathcal{O} J} \rangle_{\text{QFT}} = Z_{\text{GR}}[\phi \to J] = e^{-S_{\text{GR}}[\phi \to J]} \,.$$



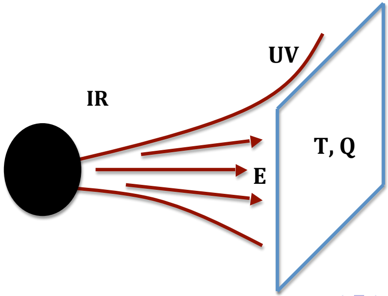
- Solve classical equation for ϕ in the bulk to obtain e.g. $\langle \mathcal{O} \rangle (J)$.

Finite temperature and charge density needs black holes

- Energy density T^{tt} dual to metric component g_{tt}. Charge density J^t dual to Maxwell component A_t.
- Unique solution with nontrivial $g_{tt}(r)$ and $A_t(r)$:

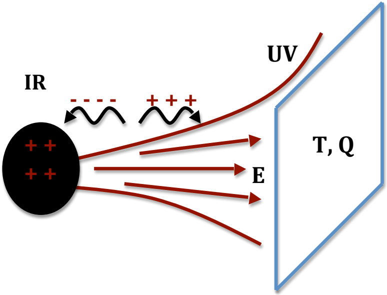
$\rightarrow$ AdS-Reissner-Nordstrom black hole

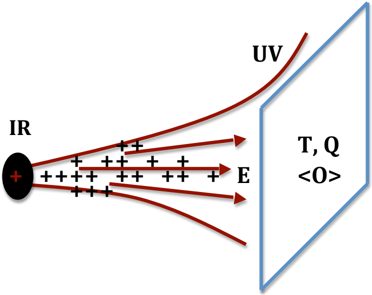
$$ds^2 = r^2\left(-f(r)dt^2 + dx^2 + dy^2\right) + \frac{dr^2}{f(r)r^2}, \quad A_t = \mu\left(1 - \frac{1}{r}\right).$$



Instabilities...

- From the black hole one can compute many properties of the 'normal' state: e.g. conductivities $\sigma(\omega, k)$, $\kappa(\omega, k)$, susceptibilities χ...
- Is this state stable? There is a natural instability in the presence of charged bulk matter. Charged scalars can lead to superconductivity.





Holographic superconductors

1. Ingredients for a holographic superconductor
2. Black hole instabilities
3. Electrical conductivity
4. Landscape of superconducting membranes

Minimal ingredients for a holographic superconductor

- Minimal ingredients
  - Continuum theory ⇒ have $T^{\mu\nu}$ ⇒ need bulk g_{ab}.
  - Conserved charge ⇒ have J^{μ} ⇒ need bulk A_a.
  - 'Cooper pair' operator ⇒ have $\mathcal{O}$ ⇒ need bulk ϕ.
- Write a minimal 'phenomenological' bulk Lagrangian

$$\mathcal{L}_{1+3} = \frac{1}{2\kappa^2} R + \frac{3}{L^2\kappa^2} - \frac{1}{4g^2} F_{ab}F^{ab} - \left|\nabla\phi - iqA\phi\right|^2 - m^2 \left|\phi\right|^2 .$$

There are four dimensionless quantities in this action.

  - Newton's constant ⇒ central charge of the CFT: $c = 192 L^2/\kappa^2$.
  - Maxwell coupling ⇒ DC conductivity $\sigma_{xx} = \frac{1}{g^2}$.
  - Mass ⇒ scaling dimension $\Delta(\Delta - 3) = (mL)^2$.
  - Charge q is the charge of the dual operator $\mathcal{O}$.

Two instabilities of a charged AdS black hole

- By dimensional analysis $T_c \propto \mu$.
- RN-AdS can be unstable against a (charged) scalar for two reasons.
- Reason 1 [Gubser '08]: Background charge shifts mass:

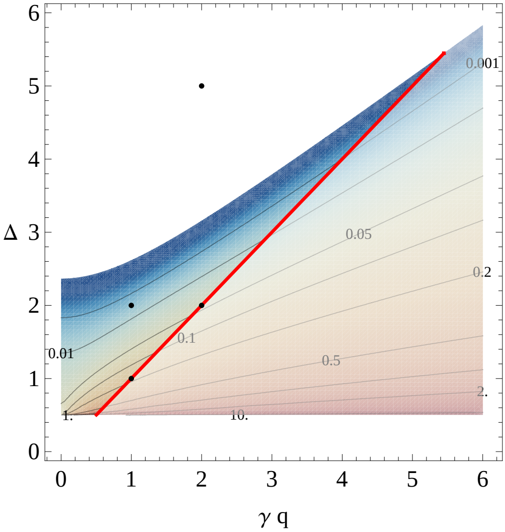
$$m^2_{\text{eff.}} \sim m^2 - q^2 A_t^2 \,.$$

- Reason 2 [SAH-Herzog-Horowitz '08]: Near extremality AdS_2 throat with

$$m^2_{\text{BF-2}} = -\frac{1}{4L_2^2} = -\frac{3}{2L^2} > -\frac{9}{4L^2} = m^2_{BF-4} \,.$$

- Precise criterion for instability at $T = 0$ [Denef-SAH '09, Gubser '08]

$$q^2\gamma^2 \geq 3 + 2\Delta(\Delta - 3) \,, \qquad \gamma^2 = \frac{2g^2 L^2}{\kappa^2} \,.$$



[Denef-SAH '09]

Endpoint – hairy black holes

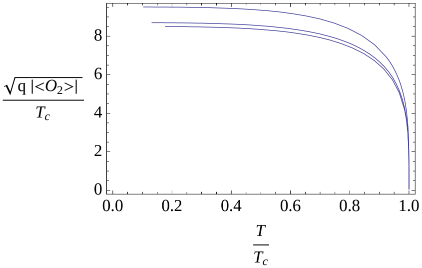
[SAH-Herzog-Horowitz '08]

- Endpoint of instability is a hairy black hole:

$$ds^2 = -g(r)e^{-\chi(r)}dt^2 + \frac{dr^2}{g(r)} + r^2\left(dx^2 + dy^2\right) ,$$

$$A_t = A_t(r) , \qquad \phi = \phi(r) .$$

- Solve numerically (take $\Delta = 2$). Can obtain $\langle \mathcal{O} \rangle$:

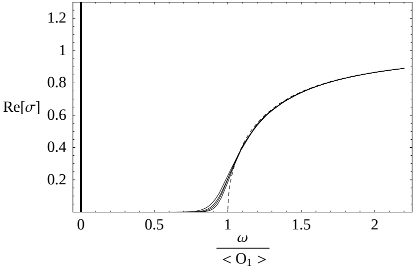


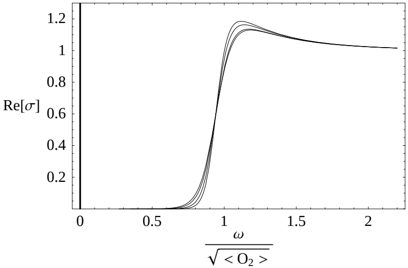
- Compare 8 to ~ 3.5 for BCS and $\sim 5 - 8$ for High-T_C.

Electrical conductivity

[SAH-Herzog-Horowitz '08]

- Computed the conductivity. At $T \sim 0$, typical curves





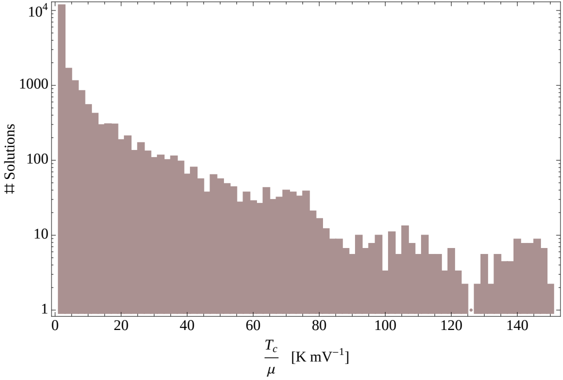
- If the gap is 2Δ then we found that

$$\mathrm{Re}\,\sigma(\omega \to 0) \sim e^{-\alpha\Delta/T}.$$

- Generally $\alpha \neq 1$, unlike BCS theory, no weakly coupled picture in terms of Cooper 'pairs'.

String landscape of superconductors
[Denef-SAH '09]

- This setup is realised in many concrete theories.
- Distribution of critical temperatures



Probing the normal state with magnetic fields

[Denef-SAH-Sachdev *in preparation*]

- The secret of superconductivity lies in the ‘normal’ state.
- The charged-AdS black hole is dual to a novel exotic state of matter.
- It is not a weakly coupled ‘Fermi liquid’.
- What is it?
- Magnetic fields are a powerful tool to characterise Fermi surfaces: de Haas-van Alphen oscillations.
- Looking for these in AdS/CFT requires one loop in the bulk ($1/N$).

Summary

- AdS/CFT: strongly coupled theories exhibiting superconductivity.
- Presence of superconductivity depends on the charge and scaling dimensions of operators in a quantum critical state.
- Showed results for condensate and electrical conductivity.
- ∃ 'landscape' of superconductors in string theory.
- Key question: how to think about the 'normal' state?